Two Simple Nonlinear Edge Detectors*

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Zhejiang University
27 May 2004

*Research supported in part by NSF grant DMS-0072234

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Introduction

Goal: Detect and characterize singularities in images by local dimensionality.

Idea: Model images and functions as random vectors. Use an autocovariance matrix as a substitute for the Jacobian, or derivative.

Application: Detect points (local dimension 0) and edges (local dimension 1) in images using the eigenvalues of the autocovariance matrix.

Prior Similar Ideas

Old Observation: if a function is rough along some direction, then its local Fourier transform stays big along that direction.

Duda and Hart [1972]: Hough transform for detection of lines and curves in images.

Mallat and Zhong [1992]: edge detection from wavelet maxima.

Aron and Kurz [1997]: linear hypothesis testing of variances in small windows to detect lines and edges.

New View: recognize nonsmooth directions from eigenvalues of an autocovariance matrix accumulated from the localized Fourier transform. Use the eigenvector of the larger eigenvalue as the normal to the edge.

Probabilistic Motivation (2-D)

Function $\phi : \mathbf{T} \to \mathbf{C}$ on the unit circle $\mathbf{T} \subset \mathbf{R}^2$.

Random direction vector $\Phi : \mathbf{T} \to \mathbf{C}^2$ is

$$\Phi(\theta) \stackrel{\text{def}}{=} \phi(\theta)(\cos\theta, \sin\theta), \qquad \theta \in \mathbf{T}.$$

Direction probability density is $|\phi|^2$, normalized.

Symmetric 2×2 Autocovariance Matrix

$$E(\Phi^*\Phi)_{ij} = \int_{\mathbf{T}} \Phi_i^* \Phi_j = \int_0^{2\pi} |\phi(\theta)|^2 \tau_i(\theta)\tau_j(\theta)\, d\theta,$$

for $i, j \in \{1, 2\}$, $\tau_1(\theta) = \sin\theta$, and $\tau_2(\theta) = \cos\theta$.

Then $\langle E(\Phi^*\Phi)v, v\rangle = \int_{\mathbf{T}} |\langle \Phi, v\rangle|^2$

– attain $\sup_{\|v\|=1} \langle E(\Phi^*\Phi)v, v\rangle$ at a unit eigenvector $v \in \mathbf{R}^2$ of the largest eigenvalue of $E(\Phi^*\Phi)$.

– Such an eigenvector v always exists.

– Use v to approximate the max of $|\phi| = \|\Phi\|$.

Example (2-D)

For $f : \mathbf{R}^2 \to \mathbf{C}$ and large fixed $R > 0$, define

$$\phi(\theta) = |\widehat{f}(R\cos\theta, R\sin\theta)|^2$$

This samples $|\widehat{f}|^2$ far out in the direction θ.

Suppose ϕ is highly concentrated near the point $\theta_0 \in \mathbf{T}$. Then $E(\Phi^*\Phi)$ is approximately proportional to

$$\begin{pmatrix} \cos^2\theta_0 & \cos\theta_0\sin\theta_0 \\ \cos\theta_0\sin\theta_0 & \sin^2\theta_0 \end{pmatrix}.$$

Any nonzero vector in the θ_0 direction is an eigenvector of the largest eigenvalue.

Alternatives

(A-1) Replace circle $\mathbf{T}$ with disc B:

$$E(\Phi^*\Phi)_{ij} = \int_B |\phi(\xi)|^2 \xi_i \xi_j \, d\xi,$$

for $i, j \in \{1, 2\}$, and $\phi \in L^2(B)$.

(A-2) Replace disk B with $\mathbf{R}^2$, under additional integrability assumptions.

Localization

Start with $f : \mathbf{R}^2 \to \mathbf{R}$, point of interest x, scale $\epsilon = 1/R$.

Localize by $f \mapsto gf$, for some nonzero radial Schwartz function $g : \mathbf{R}^2 \to \mathbf{R}$:

$$g_\epsilon(y) = g\left(\frac{y-x}{\epsilon}\right), \qquad \epsilon > 0.$$

which is:

- smooth, to avoid introducing new singularities;
- radial, to avoid introducing directional bias;
- nonzero and concentrated, to emphasize the point of interest x.

Dual Local Autocovariance

Definition 1 *The* dual local autocovariance matrix of f at x *is the* 2×2 *matrix whose* ij*-coefficient is:*

$$E_{\epsilon,g}(f;x)_{ij} = \int_{B(0,1/\epsilon)} \xi_i \xi_j \left|\widehat{g_\epsilon f}(\xi)\right|^2 \, d\xi.$$

This is the real, symmetric second moment matrix of the unnormalized probability density function $|\widehat{g_\epsilon f}|^2$.

Localization gf is integrable $\Rightarrow$ $\widehat{gf}$ is bounded and continuous $\Rightarrow$ matrix coefficients are well defined.

$f \not\equiv 0$ near $x \Rightarrow E_{\epsilon,g}(f;x)$ is positive definite.

Straight Edges

Change of variables: $E_{\epsilon,g}(f;x)_{ij} =$

$$\int_B \xi_i \xi_j \left| \int_{\mathbf{R}^2} g\left(y + \frac{\epsilon - 1}{\epsilon} x\right) f(\epsilon y) e^{-2\pi i y \cdot \xi}\, dy \right|^2 d\xi.$$

If f is homogeneous of degree 0, and $x = 0$, this formula is independent of $\epsilon > 0$

Example: $f = \mathbf{1}_L$, indicator function of left half-plane $L = \{(x_1, x_2) : x_1 \leq 0\}$.

– $E_{\epsilon,g}(\mathbf{1}_L; 0)$ is a matrix with no ϵ-dependence.

– Use $g(x) = \exp(-\pi|x|^2)$, for ease of computing Fourier transforms.

– Get dual local autocovariance matrix explicitly for the edge $x_1 = 0$.

Straight edges (continued. . .)

Lemma 1 $E_{\epsilon,g}(1_L; 0)$ *has the eigensystem*

$$\lambda_1 \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \lambda_2 \leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

satisfying $\lambda_1 > \lambda_2 > 0$, *independently of* ϵ.

Numerical estimate: the matrix is diagonal, so

$$\lambda_i = \int_B \xi_i^2 \left| e^{-\pi\xi_2^2} \int_{-\infty}^{0} e^{-\pi x^2 - 2\pi i x \xi_1}\, dx \right|^2 d\xi,$$

for $i = 1, 2$. In terms of the Gaussian integral $G(\xi) = \int_0^\xi \exp(\pi x^2)\, dx$, this becomes:

$$\lambda_i = \int_B \xi_i^2 e^{-2\pi|\xi|^2} \left[\frac{1}{4} + G^2(\xi_1) \right] d\xi,$$

for $i = 1, 2$.

By *Mathematica*, $\lambda_1 - \lambda_2 \approx 0.02258364$.

Off the Straight Edge

Conversely, take $g(\xi)$ to be a smooth radial function supported in $B(0,1)$, so:

– for any $x \notin L$, and $0 < \epsilon < \text{dist}\,(L, x)$ small enough,

$$E_{\epsilon,g}(\mathbf{1}_L; x) \equiv 0,$$

since $g_\epsilon \mathbf{1}_L = 0$.

– for each point $x \in \text{Int}L$,

$$E_{\epsilon,g}(\mathbf{1}_L; x) = const \cdot I,$$

for all sufficiently small $0 < \epsilon < \text{dist}\,(x, \partial L)$.

In either case, E has two equal eigenvalues.

Rotation and Translation

Translation moves x, preserves eigenvectors:

Lemma 2 *Let T be a translation by x in $\mathbf{R}^2$: $f \circ T(y) = f(y + x)$. Then*

$$E_{\epsilon,g}(f; x) = E_{\epsilon,g}(f \circ T; 0).$$

Rotation about x rotates eigenvectors, changes edge direction:

Lemma 3 *Let U be a rotation about x in $\mathbf{R}^2$. Then*

$$E_{\epsilon,g}(f \circ U; x) = U \circ E_{\epsilon,g}(f; x) \circ U^{-1}.$$

Straight Edge Detection

Let $\mathbf{1}_H$ be the characteristic function of a half-plane

$$H = \{x \in \mathbf{R}^2 : \nu \cdot x \leq \beta\},$$

with nonzero normal vector $\nu \in \mathbf{R}^2$, for some constant $\beta \in \mathbf{R}$. Its *edge* is the line ∂H.

Theorem 4 *$x \in \partial H$ iff $E_{\epsilon,g}(\mathbf{1}_H; x)$ has distinct eigenvalues $\lambda_1 > \lambda_2 > 0$ for every smooth, radial function $g \not\equiv 0$, and $\epsilon > 0$. In that case, ν will be an eigenvector of the larger eigenvalue.*

Note; degree-0 homogeneity of $\mathbf{1}_H \Rightarrow$ eigenvalues of $E_{\epsilon,g}(\mathbf{1}_H; x)$ do not depend on ϵ.

For $x \notin \partial H$ off the edge, and each smooth, compactly-supported radial function g,

$$E_{\epsilon,g}(\mathbf{1}_H; x) = const \cdot I$$

has eventually constant and equal eigenvalues as $\epsilon \to 0$.

Smooth-Curve Edges

Idea: Boundaries of smooth domains look like boundaries of half-planes.

Fix:

– Domain $D \subset \mathbf{R}^2$, smooth boundary ∂D.

– $\mathbf{1}_D$, the characteristic function of D.

– $g : \mathbf{R}^2 \to \mathbf{R}$, a smooth radial function, supported in $B = B(0, 1)$.

– H, half-plane with $\partial H \parallel \partial D$ at some x.

As $\epsilon \to 0$, expect:

– $x \in \partial D \Rightarrow$ distinct eigenvalues for $E_{\epsilon,g}(\mathbf{1}_D, x)$.

– $x \notin \partial D \Rightarrow$ equal eigenvalues for $E_{\epsilon,g}(\mathbf{1}_D, x)$.

Smooth-Curve Edges (continued. . .)

Lemma 5 *In any matrix norm,*

$$\|E_{\epsilon,g}(\mathbf{1}_H; x) - E_{\epsilon,g}(\mathbf{1}_D; x)\| = O(\epsilon),$$

as $\epsilon \to 0$.

(Note: for Schwartz g, any $\delta > 0$ gives

$$\|E_{\epsilon,g}(\mathbf{1}_H; x) - E_{\epsilon,g}(\mathbf{1}_D; x)\| = O(\epsilon^{1-\delta}),$$

as $\epsilon \to 0$.)

Theorem 6 *For* $x \in \partial D$, *and eigenvalues* $\lambda_1(\epsilon)$ *and* $\lambda_2(\epsilon)$ *of* $E_{\epsilon,g}(\mathbf{1}_D; x)$, *we have*

$$\liminf_{\epsilon \to 0+} |\lambda_1(\epsilon) - \lambda_2(\epsilon)| > 0.$$

For $x \notin \partial D$, $|\lambda_1(\epsilon) - \lambda_2(\epsilon)| \to 0$ *as* $\epsilon \to 0$.

Affine and Differentiable Functions

At points x where f is differentiable, the eigenvalues $\lambda_1(\epsilon), \lambda_2(\epsilon)$ of $E_{\epsilon,g}(f;x)$ are equal in the limit:

Lemma 7 *Let $A : R^2 \to \mathbf{R}$ be an affine function: $A(x) = a \cdot x + b$ for some constants $a \in \mathbf{R}^2, b \in \mathbf{R}$. Let $g : \mathbf{R}^2 \to \mathbf{R}$ be a smooth radial function supported in $B = B(0,1)$. Then:*

$$\lim_{\epsilon \to 0+} [\lambda_1(\epsilon) - \lambda_2(\epsilon)] = 0.$$

Lemma 8 *If $f : \mathbf{R}^2 \to \mathbf{R}$ is differentiable at x with affine A tangent to f at x, then*

$$\|E_{\epsilon,g}(f;x) - E_{\epsilon,g}(A;x)\| = o(\epsilon),$$

as $\epsilon \to 0$.

(Continuously differentiable f near x satisfies

$$\|E_{\epsilon,g}(f;x) - E_{\epsilon,g}(A;x)\| = O(\epsilon^2),$$

as $\epsilon \to 0$.)

Detecting Non-Edge Points

Theorem 9 *Suppose that* $f : \mathbf{R}^2 \to \mathbf{R}$ *is differentiable at* x. *Then for any smooth radial compactly-supported function* $g : \mathbf{R}^2 \to \mathbf{R}$,

$$E_{\epsilon,g}(f; x) \to const \cdot I,$$

as $\epsilon \to 0$.

Converse is false: $\lim_{\epsilon \to 0+} |E_{\epsilon,g}(f; x)| = const \cdot I$ does not imply that f is differentiable at x, or even continuous. Symmetry can masquerade as smoothness, as in $f = \mathbf{1}_{x_1,x_2>0}$. Fix $g(x) = \exp(-\pi|x|^2)$ as before. Then

$$E_{\epsilon,g}(f; 0) = const \cdot I \neq 0$$

for every $\epsilon > 0$, so $\lambda_1(\epsilon) = \lambda_2(\epsilon) = \lambda > 0$ for every ϵ, even though f is discontinuous at 0.

Higher-Dimensional Theory

Study the geometry of $f : \mathbf{R}^p \to \mathbf{R}^d$ in high dimensions p, d.

Suppose $f = (f_1, \ldots, f_d)$ is polynomially bounded. Fix $x \in \mathbf{R}^p$, Schwartz $g : \mathbf{R}^p \to \mathbf{R}^d$ with radial components, and $g_\epsilon(y) \stackrel{\text{def}}{=} g\left(\frac{y-x}{\epsilon}\right)$, as before, for $\epsilon > 0$.

Definition 2 *The* dual local autocovariance matrix of f at x *is the* $p \times p$ *matrix:*

$$E_{\epsilon,g}(f; x)_{ij} = \int_{B(0,1/\epsilon)} \xi_i \xi_j \|\widehat{f \cdot g_\epsilon}(\xi)\|^2 \, d\xi,$$

for $i, j \in \{1, \ldots, p\}$.

Smooth $p \times p$ case is like smooth 2×2:

Theorem 10 *Suppose that* $f : \mathbf{R}^p \to \mathbf{R}^d$ *is differentiable at* x. *Then for any smooth radial function* $g : \mathbf{R}^p \to \mathbf{R}^d$ *of compact support, the matrix* $E_{\epsilon,g}(f; x) \to const \cdot I$, *as* $\epsilon \to 0$.

Discretization

Discrete Fourier transform on N real samples $\{f(n) : 0 \le n < N\}$:

$$\widehat{f}(k) = \sum_{n=0}^{N-1} \exp\left(-2\pi i \frac{kn}{N}\right) f(n),$$

for integer $k \in B_N = \left[-\frac{N}{2}, \frac{N}{2}\right]$.

If only the first $q \ll N$ samples of f are nonzero, then the sum is over $\{0, 1, \ldots, q-1\}$.

When f is real-valued,

$$|\widehat{f}(k)|^2 = \sum_{n,n'=0}^{q-1} \exp\left(-2\pi i \frac{k(n-n')}{N}\right) f(n) f(n'),$$

for $k \in B_N$.

Discretization (continued. . .)

The r-th moment of $|\widehat{f}(k)|^2$ is $\sum_{k\in B_N} k^r |\widehat{f}(k)|^2 =$

$$\sum_{n,n'=0}^{q-1} f(n)f(n') \sum_{k\in B_N} k^r \exp\left(-2\pi i \frac{k(n-n')}{N}\right).$$

The innermost sum in k, if normalized with N^{r+1}, is a Riemann sum for the integral

$$\mu_r(n) = \int_{-\frac{1}{2}}^{\frac{1}{2}} x^r \exp(-2\pi i n x)\, dx,$$

evaluated at $n \leftarrow (n-n')$:

$$\mu_0(n) = \begin{cases} 1, & \text{if } n = 0, \\ 0, & \text{otherwise;} \end{cases}$$

$$\mu_1(n) = \begin{cases} 0, & \text{if } n = 0, \\ i\frac{(-1)^n}{2\pi n}, & \text{otherwise;} \end{cases}$$

$$\mu_2(n) = \begin{cases} 1/12, & \text{if } n = 0, \\ \frac{(-1)^n}{2\pi^2 n^2}, & \text{otherwise.} \end{cases}$$

Discrete Dual Local Autocovariance

Image: f on $\{(m,n) : 0 \le m < M;\ 0 \le n < N\}$.

Bump: g on $\{(m,n) : 0 \le m < p; 0 \le n < q\}$, for $p \ll M; q \ll N$.

Autocovariance matrix: localize $f \leftarrow gf$, then compute

$$E_{11} = \sum_{m,m'=0}^{p-1} \sum_{n=0}^{q-1} f(m,n) f(m',n) \mu_2(m-m');$$

$$E_{22} = \sum_{m=0}^{p-1} \sum_{n,n'=0}^{q-1} f(m,n) f(m,n') \mu_2(n-n');$$

$$E_{12} = E_{21} =$$

$$\sum_{m,m'=0}^{p-1} \sum_{n,n'=0}^{q-1} f(m,n) f(m',n') \mu_1(m-m') \mu_1(n-n').$$

Implementation in General

Around each point z of the image, do:

Localization. Extract pixel values on the square subgrid contained in $z + [-\epsilon, \epsilon]^2$. Multiply $f(z+y)$ by the weighting bump function $g(y)$. [$O(\epsilon^2)$ operations per pixel.]

Dual autocovariance. Compute $E = (E_{ij})$, a real-valued, symmetric, positive semidefinite 2×2 matrix. [$O(\epsilon^4)$ operations per pixel.]

Eigenvalues. For symmetric 2×2 matrices E:

$$\lambda = \frac{1}{2}\left(E_{11} + E_{22} \pm \sqrt{(E_{11} - E_{22})^2 + 4E_{12}^2}\right),$$

where $+$ gives λ_1, $-$ gives λ_2. [$O(1)$ operations per pixel.]

Edginess. Compute ratio λ_1/λ_2 or difference $\lambda_1 - \lambda_2$, or use reciprocal $\lambda_2/\lambda_1 \in [0,1]$ in write-black mode so darker $\Rightarrow$ edgier. [$O(1)$ operations per pixel.]

Special 7x7 "Gridpoint" Case

Let z be of the form (m, n), a "gridpoint" with integer coordinates.

Use $\epsilon = 3$, to localize to 7×7 subgrids.

Zero-pad at boundaries.

Let g be the Gaussian bump $g(y) = \exp(-\pi|y|^2/\epsilon^2)$.

Use reciprocal edginess $e = \lambda_2/\lambda_1 \in [0, 1]$ in write-black mode so darker $\Rightarrow$ edgier.

Special 2x2 "Midpoint" Case

Let z be of the form $(m + \frac{1}{2}, n + \frac{1}{2})$, a "midpoint."

Let $\epsilon = 1$, so E is computed from the pixels immediately NE, SE, SW, and NW of z:

$$\begin{array}{ccc} (m, n+1) & & (m+1, n+1) \\ & z & \\ (m, n) & & (m+1, n) \end{array} \quad \leftrightarrow \quad \begin{array}{cc} NW \stackrel{\text{def}}{=} & NE \stackrel{\text{def}}{=} \\ f(m, n+1) & f(m+1, n+1) \\ & [z] \\ SW \stackrel{\text{def}}{=} & SE \stackrel{\text{def}}{=} \\ f(m, n) & f(m+1, n) \end{array}$$

Use eigenvalue difference for edginess:

$$e \stackrel{\text{def}}{=} \lambda_1(\epsilon, z) - \lambda_2(\epsilon, z),$$

All pixels are equidistant from z, so normalize $g = 1$ at that distance.

Special 2x2 "Midpoint" Case (continued...)

Evaluate $\mu_1(0) = 0$, $\mu_1(\pm 1) = \frac{\pm 1}{2\pi i}$, $\mu_2(0) = \frac{1}{12}$,and $\mu_2(\pm 1) = \frac{-1}{2\pi^2}$, so

$$\begin{aligned}
E_{11} &= (NW^2 + NE^2)\mu_2(0-0) + (SW^2 + + SE^2)\mu_2(1-1) + \\
&\qquad + (NW \cdot SW + NE \cdot SE)\,(\mu_2(0-1) + \mu_2(1-0)) \\
&= \frac{1}{12}\left(NW^2 + NE^2 + SW^2 + SE^2\right) - \frac{1}{\pi^2}\,(NW \cdot SW + NE \cdot SE)\,; \\
E_{22} &= \frac{1}{12}\left(NW^2 + NE^2 + SW^2 + SE^2\right) - \frac{1}{\pi^2}\,(NW \cdot NE + SW \cdot SE)\,; \\
\Rightarrow E_{11} - E_{22} &= \frac{1}{\pi^2}\,(NW \cdot NE + SW \cdot SE - NW \cdot SW - NE \cdot SE) \\
&= \frac{1}{\pi^2}\,(NW - SE)\,(NE - SW)\,; \\
E_{12} &= E_{21} = -\frac{1}{2\pi^2}\,(NW \cdot SE - NE \cdot SW)\,.
\end{aligned}$$

Edginess formula is a nonlinear four-tap filter with a square 2×2 pixel mask:

$$e = \lambda_1 - \lambda_2 = \frac{1}{\pi^2}\sqrt{(NW-SE)^2\,(NE-SW)^2 + (NW \cdot SE - NE \cdot SW)^2}.$$

Special 4+1 "Centerpoint" Case

Let z be of the form (m, n).

Let $\epsilon = 1$, so E is computed from the central pixel at $z = (m, n)$ and the 4 pixels N, E, S, and W of z:

$$\begin{array}{ccccc} & & (m, n{+}1) & & \\ (m{-}1, n) & z = (m, n) & (m{+}1, n) & \leftrightarrow & \begin{array}{ccc} & N \stackrel{\text{def}}{=} f(m, n{+}1) & \\ W \stackrel{\text{def}}{=} f(m{-}1, n) & C \stackrel{\text{def}}{=} f(m, n) & E \stackrel{\text{def}}{=} f(m{+}1, n) \\ & S \stackrel{\text{def}}{=} f(m, n{-}1) & \end{array} \\ & (m, n{-}1) & & & \end{array}$$

Use eigenvalue difference for edginess:

$$e \stackrel{\text{def}}{=} \lambda_1(\epsilon, z) - \lambda_2(\epsilon, z),$$

Use g with $g(0) = 1$ and $g(1) = t > 0$, so z gets 1 while N,E,S,W get t.

Special 4+1 "Centerpoint" Case (continued...)

Evaluate $\mu_1(0) = 0$, $\mu_1(\pm 1) = \frac{\pm 1}{2\pi i}$, $\mu_1(\pm 2) = \frac{\pm i}{4\pi}$, $\mu_2(0) = \frac{1}{12}$, $\mu_2(\pm 1) = \frac{-1}{2\pi^2}$, and $\mu_2(\pm 2) = \frac{1}{8\pi^2}$:

$$\begin{aligned}
D &\stackrel{\text{def}}{=} \frac{1}{12}(C^2 + t^2[N^2 + E^2 + S^2 + W^2]); \\
E_{11} &= D - \frac{t}{\pi^2}(N \cdot C + C \cdot S) + \frac{t^2}{4\pi^2}(N \cdot S); \\
E_{22} &= D - \frac{t}{\pi^2}(E \cdot C + C \cdot W) + \frac{t^2}{4\pi^2}(E \cdot W); \\
E_{12} = E_{21} &= \frac{t^2}{2\pi^2}(N \cdot W - N \cdot E + S \cdot E - S \cdot W) = \frac{t^2}{2\pi^2}(N - S)(E - W).
\end{aligned}$$

Edginess formule is a nonlinear five-tap filter with a mask shaped like a plus sign:

$$\lambda_1 - \lambda_2 = \frac{t}{\pi^2}\sqrt{\left[C\,(E+W-N-S) + \frac{t}{4}\,(N \cdot S - E \cdot W)\right]^2 + t^2\,(N-S)^2\,(E-W)^2}.$$

Each weight $t \in (0, 1)$ tunes a different filter. Use $t = 0.75$ for no good reason. The choice $t = 0$ gives $e = 0$ everywhere.

Example Images

1. Geometrical figures, piecewise constant functions with jump discontinuities along rectifiable, mostly smooth curves.

2. Toy house demonstration image from the XHoughTool website.

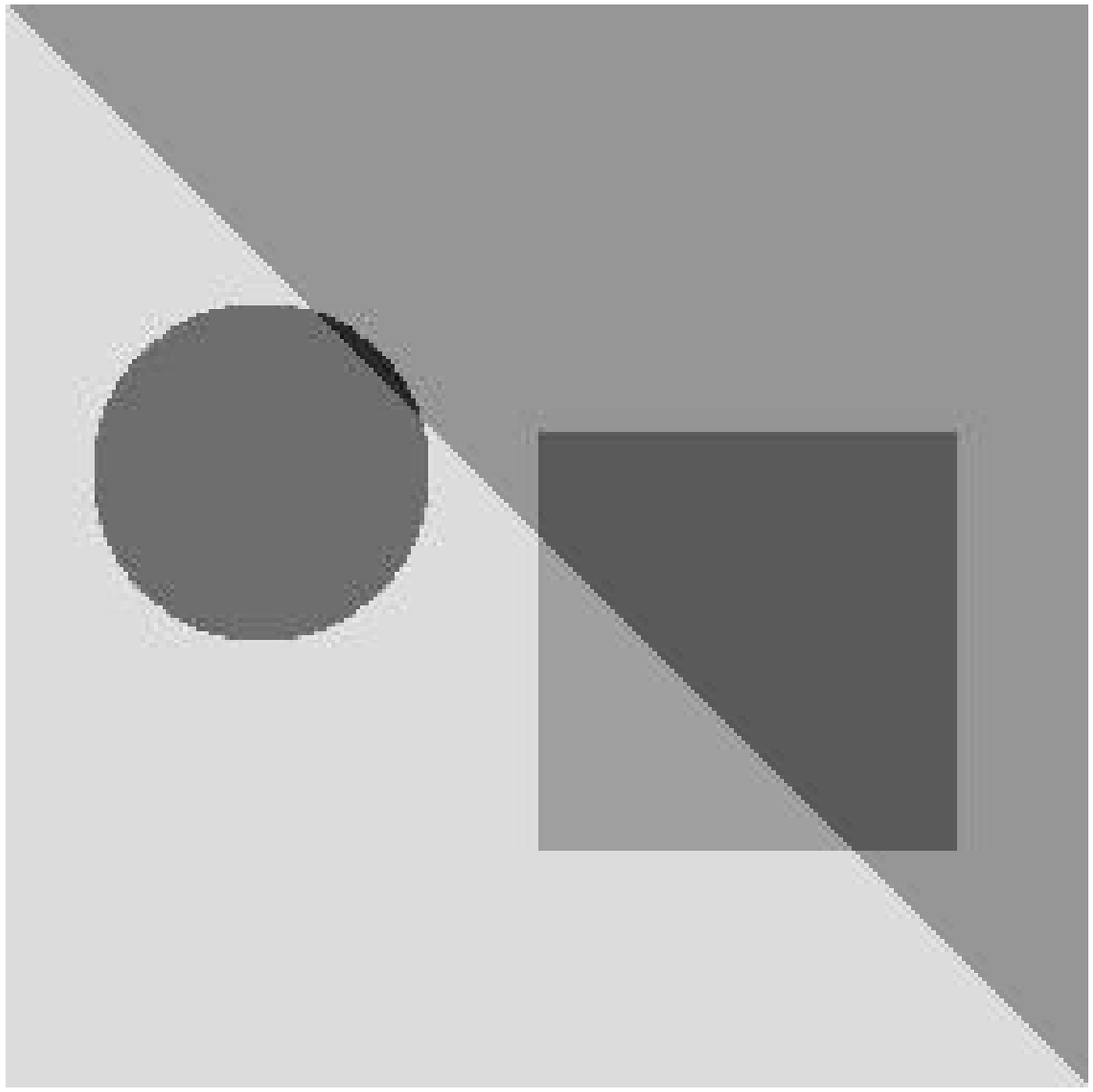
3. Fingerprint, part of the FBI/NIST compliance test suite for WSQ compression.

4. Lena Sjøblum, the famous woman in a hat.

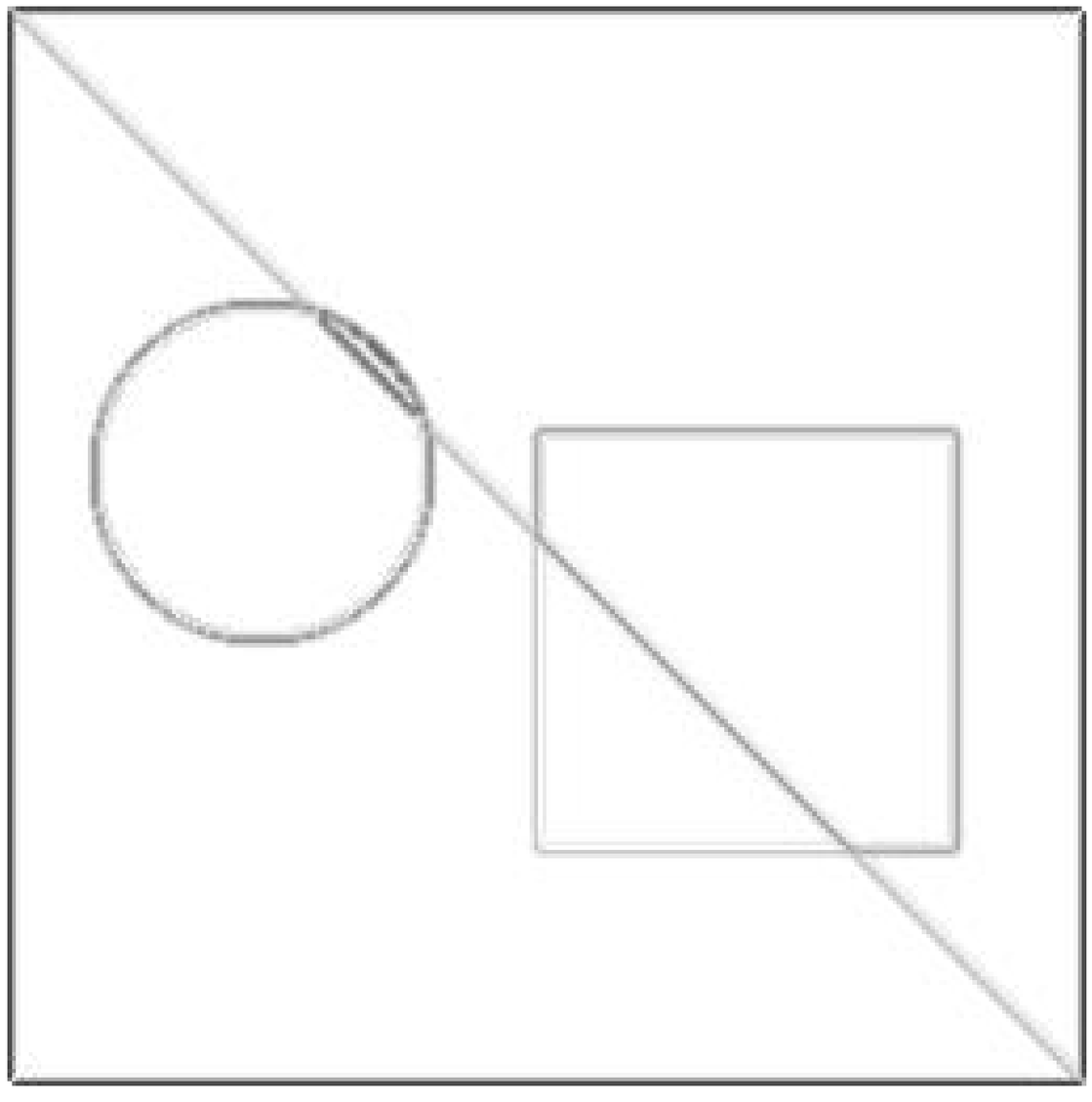
5. Cone, a ray-traced image by Craig Kolb.

6. Truck, one frame of video from Peng Li.

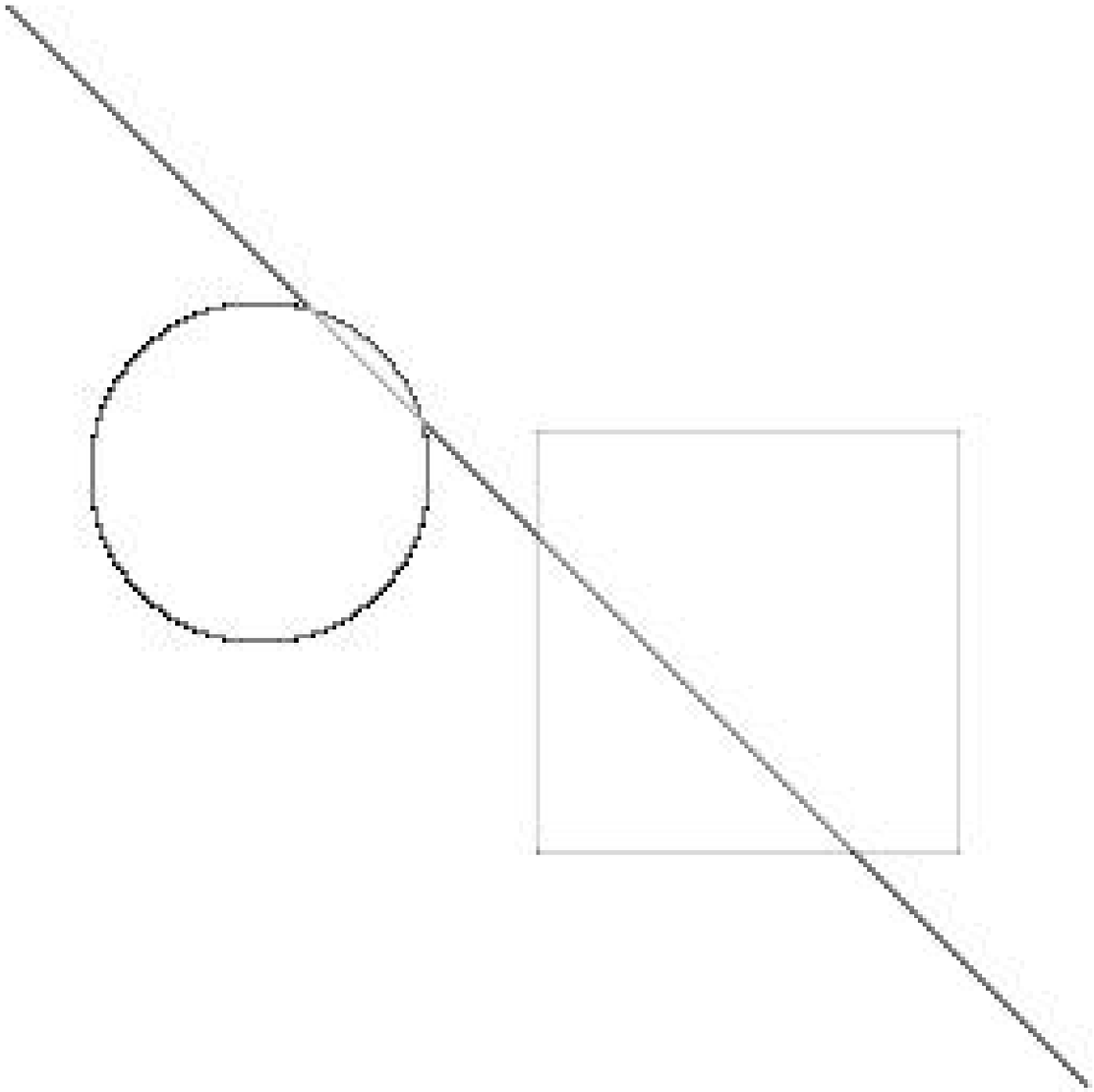
Example: Geometrical Figures



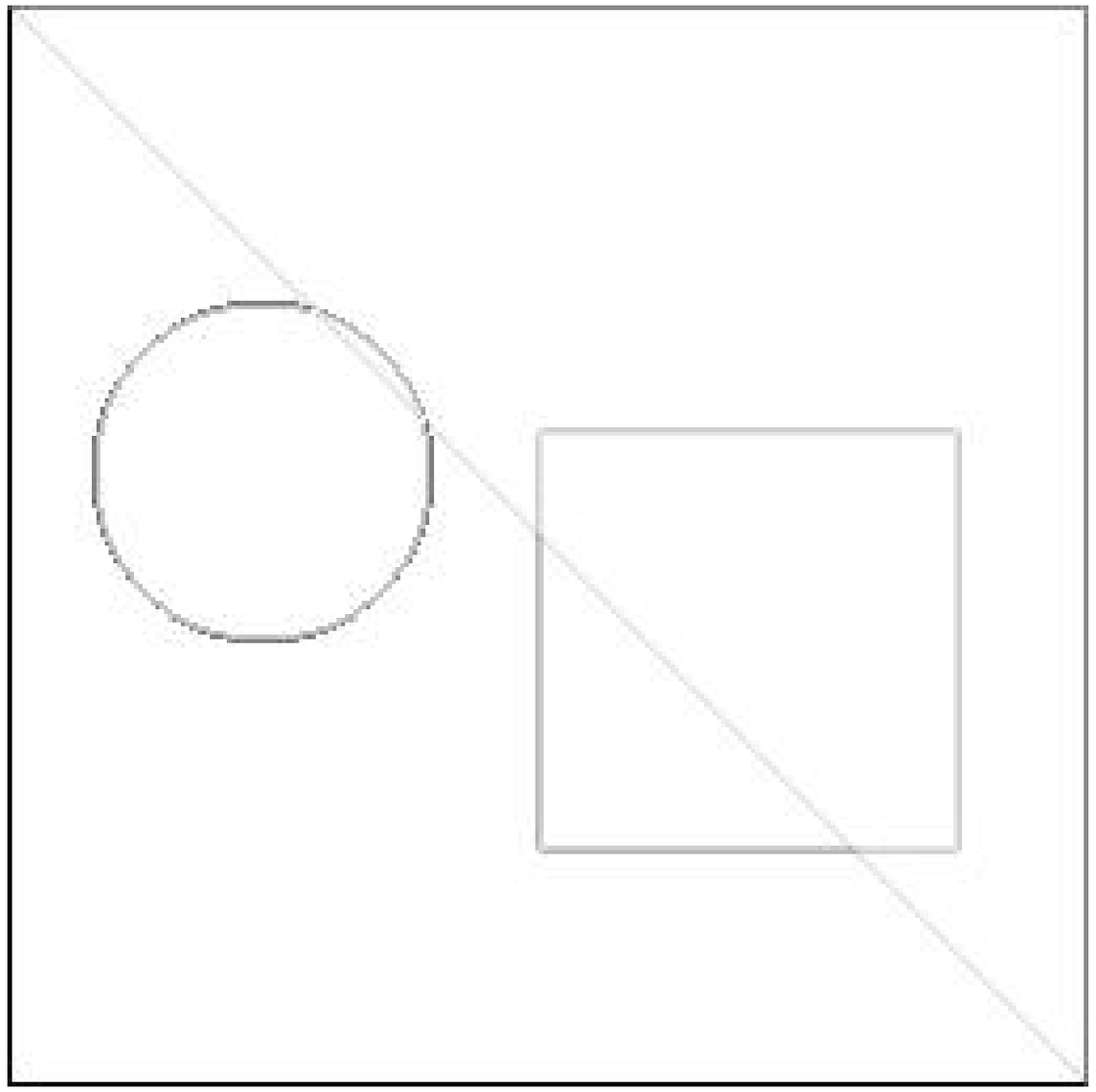
Geometrical Figures 7x7 “Gridpoint” Edginess



Geometrical Figures 2x2 “Midpoint” Edginess



Geometrical Figures 4+1 “Centerpoint” Edginess



Example: Toy House Image



Toy House 7x7 “Gridpoint” Edginess



Toy House 2x2 “Midpoint” Edginess



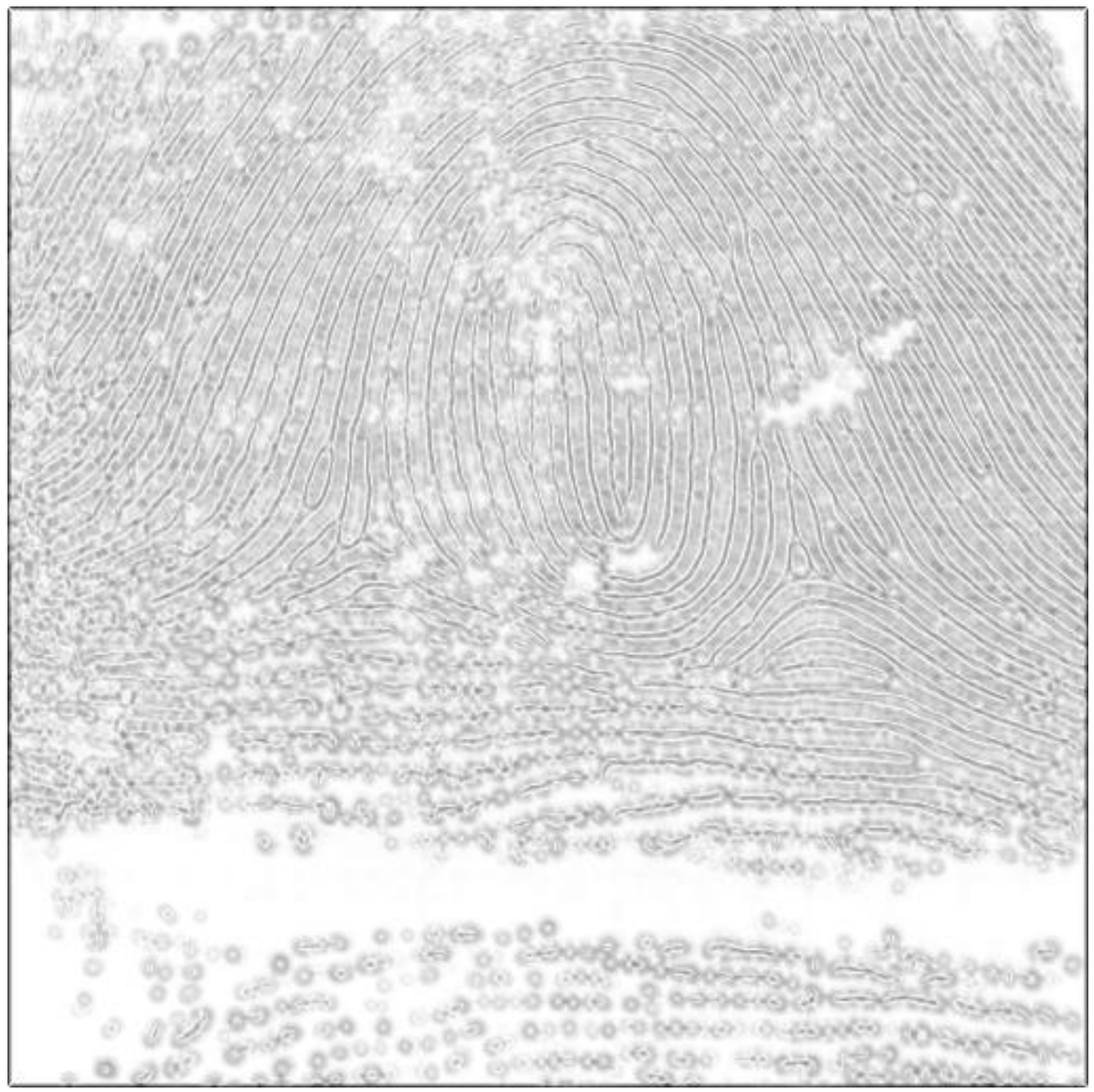
Toy House 4+1 “Centerpoint” Edginess



Example: Fingerprint Image



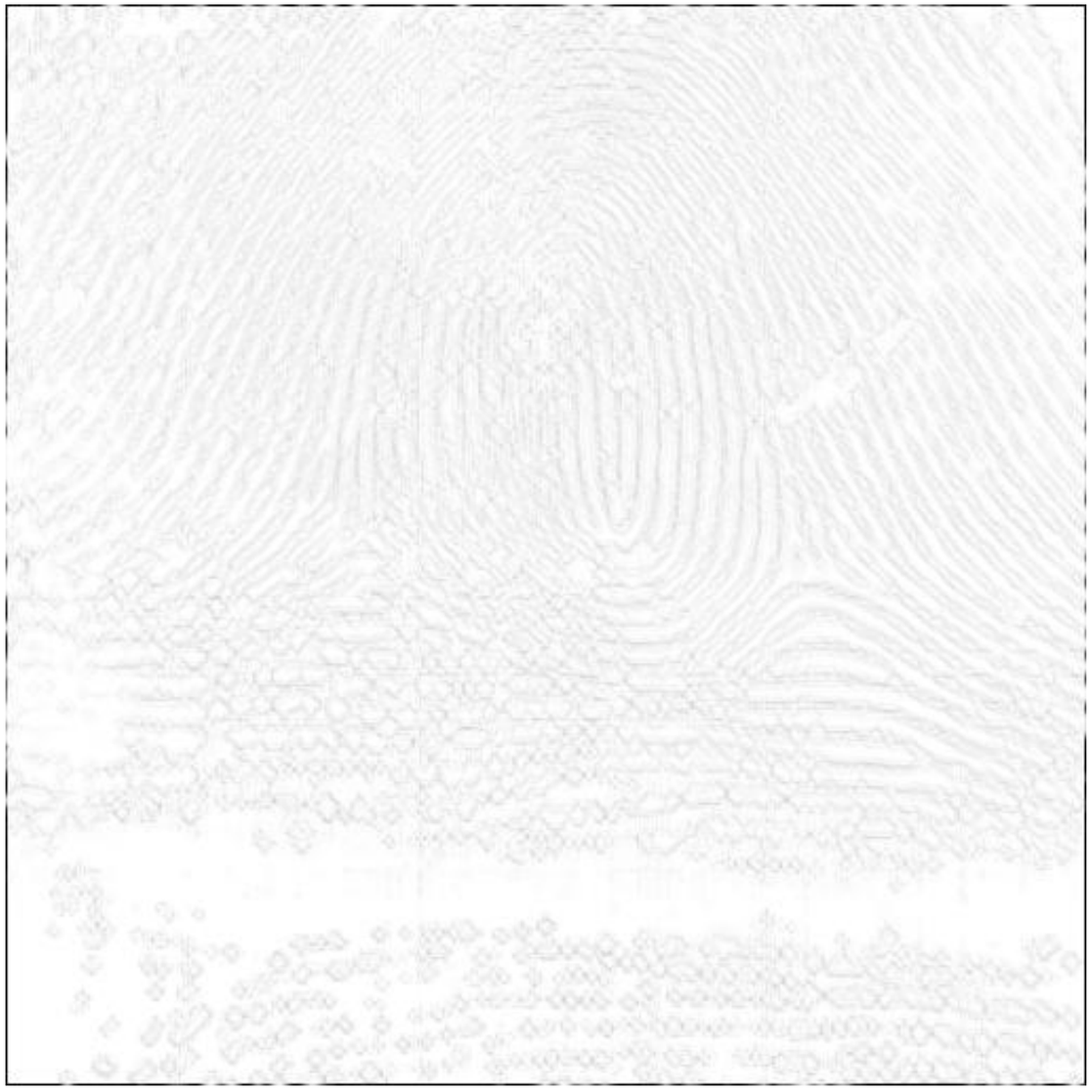
Fingerprint 7x7 “Gridpoint” Edginess



Fingerprint 2x2 “Midpoint” Edginess



Fingerprint 4+1 “Centerpoint” Edginess



Example: Lena Sjøblum Image



Lena 7x7 “Gridpoint” Edginess



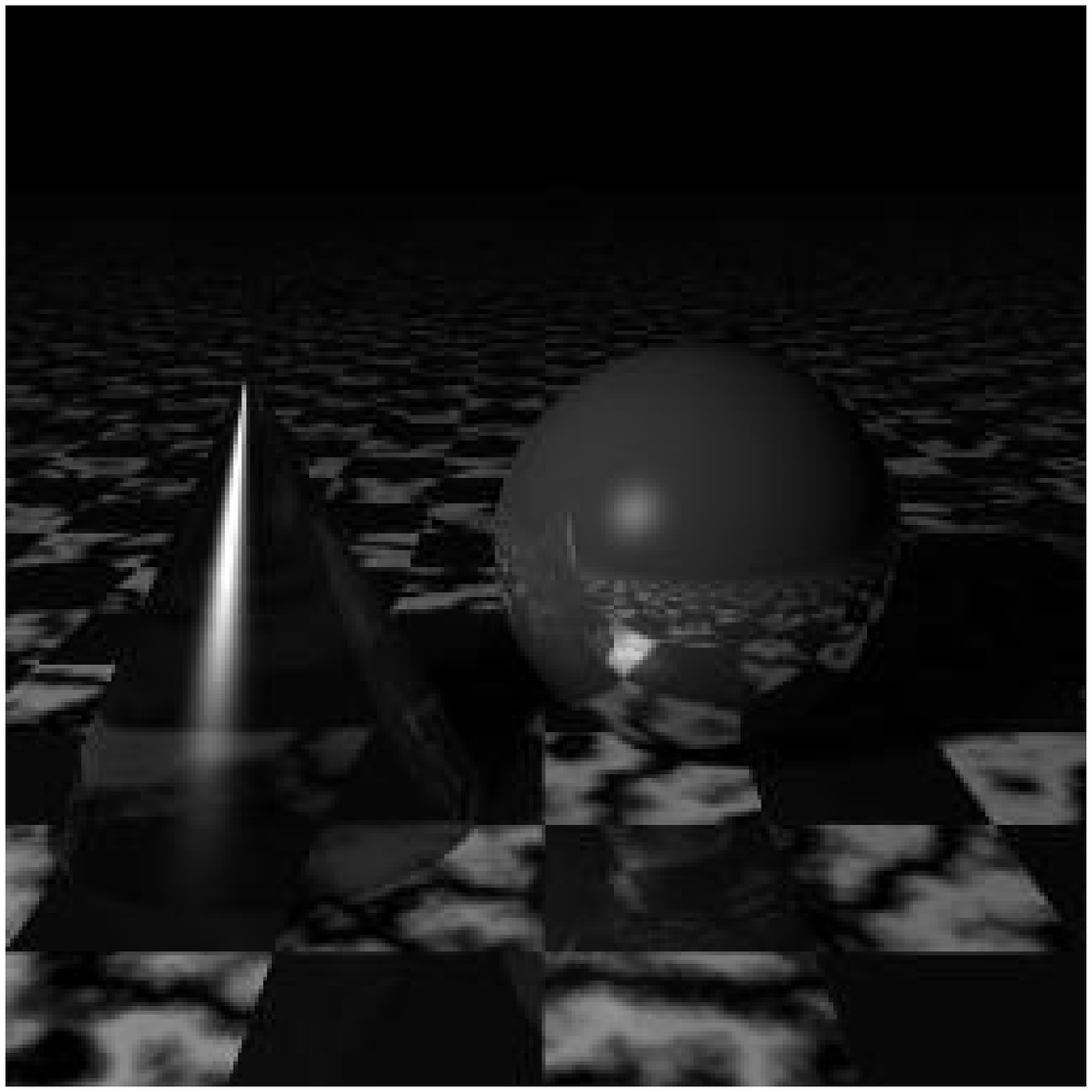
Lena 2x2 “Midpoint” Edginess



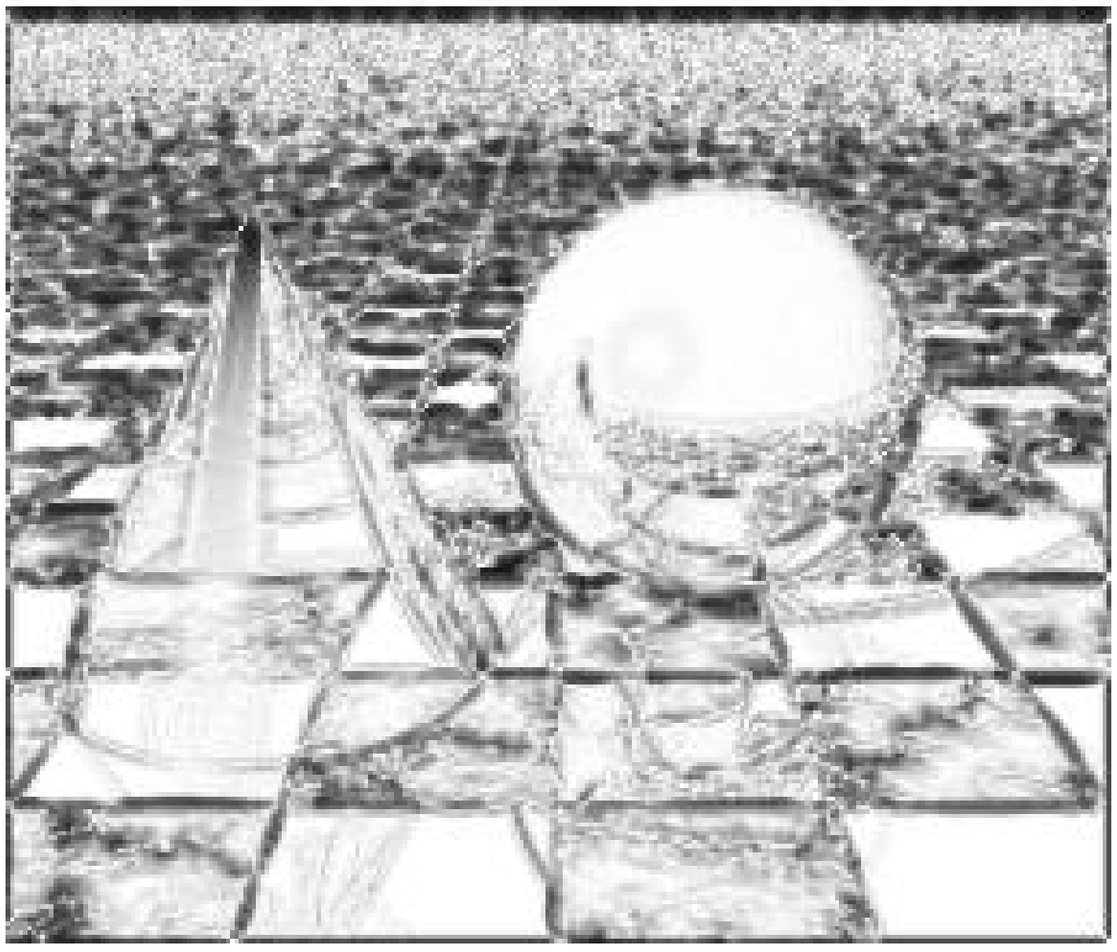
Lena 4+1 “Centerpoint” Edginess



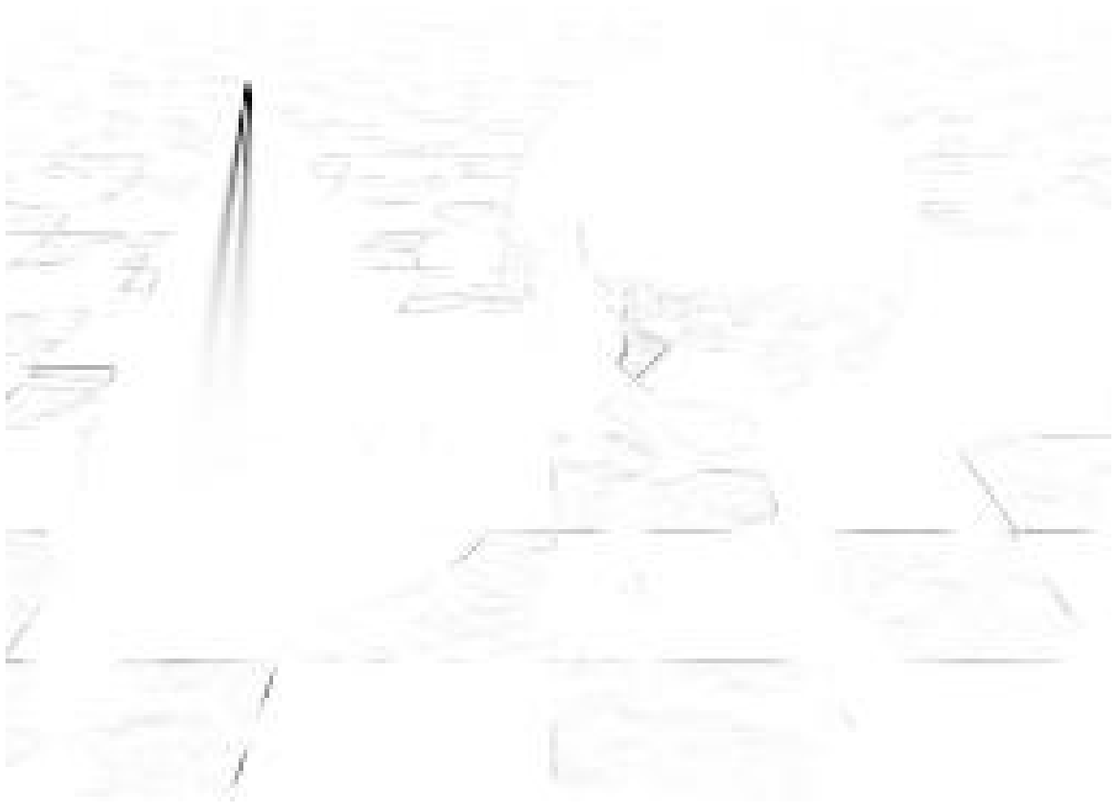
Example: Cone Image



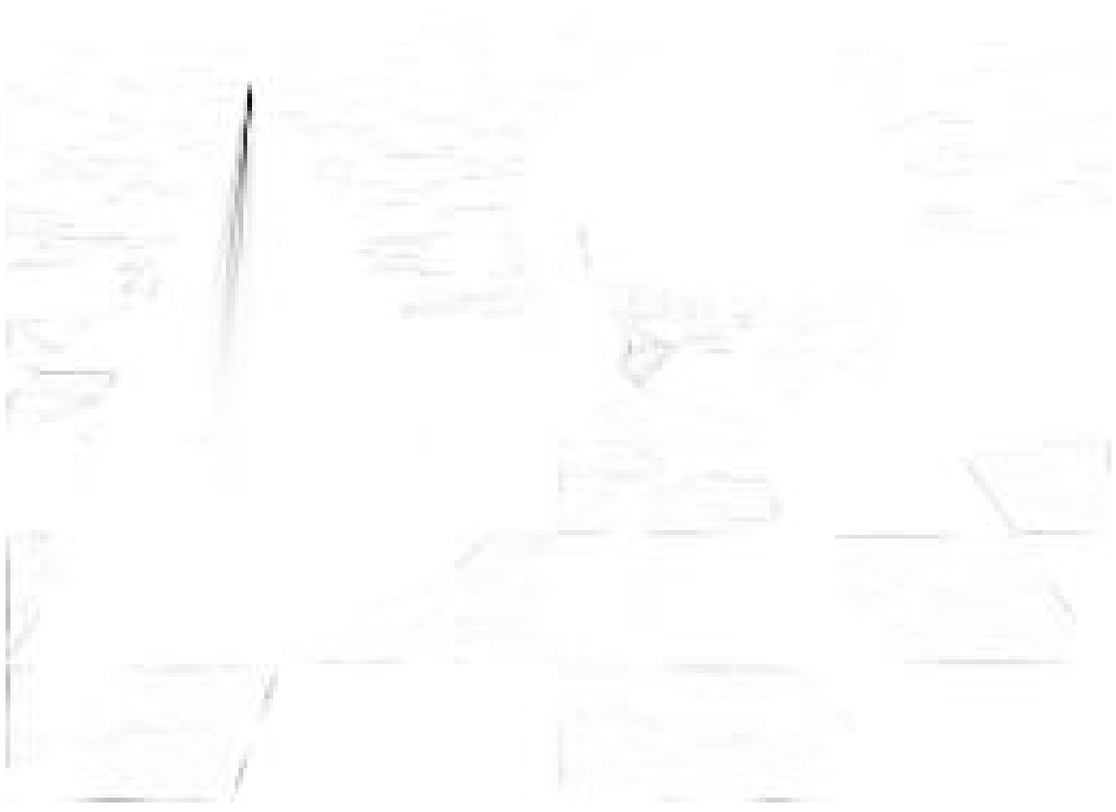
Cone 7x7 "Gridpoint" Edginess



Cone 2x2 “Midpoint” Edginess



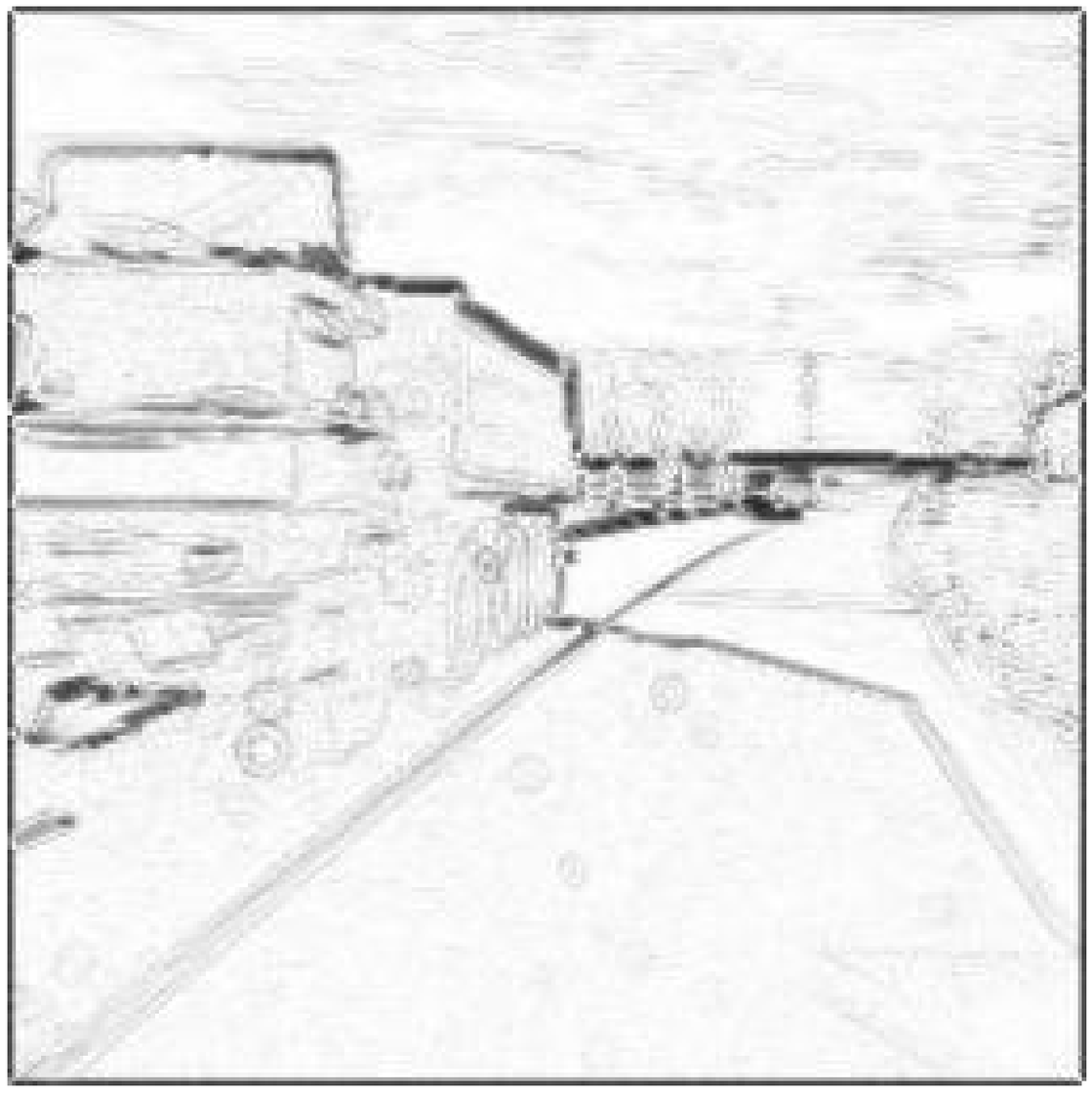
Cone 4+1 “Centerpoint” Edginess



Example: Truck Video Frame



Truck 7x7 “Gridpoint” Edginess



Truck 2x2 “Midpoint” Edginess



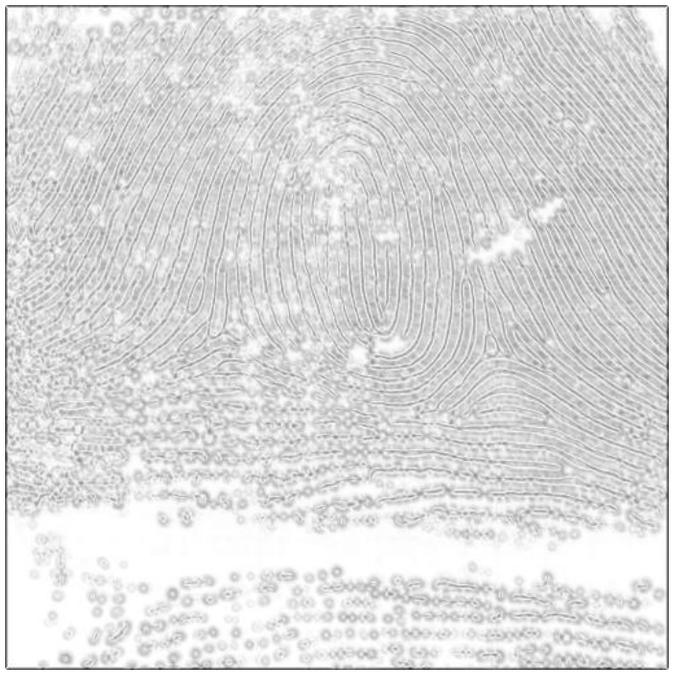
Truck 4+1 “Centerpoint” Edginess



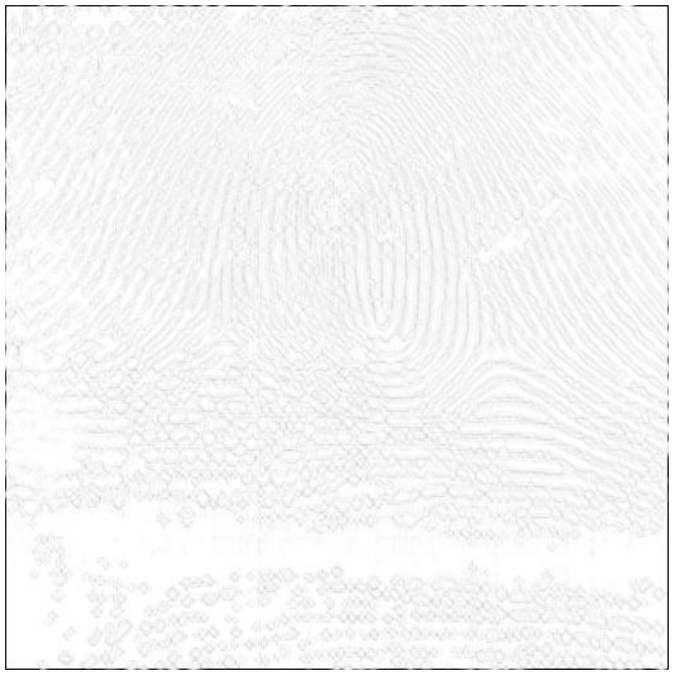
Comparison: Toy House Image 7x7, 2x2, 4+1 Edginess



Comparison: Fingerprint Image 7x7, 2x2, 4+1 Edginess







Comparison: Lena Sjøblum Image 7x7, 2x2, 4+1 Edginess







Comparison: Cone Image 7x7, 2x2, 4+1 Edginess



Comparison: Truck Video Frame 7x7, 2x2, 4+1 Edginess

