Math 456: Introduction to Financial Mathematics

Homework Set 3

Note: You may assume that the probabiliy mass function is never zero in any finite probability space Ω, namely that $\Pr(\{\omega\}) > 0$ for every point $\omega \in \Omega$.

1. Suppose that $S(t,\omega)$, $0 \le t \le T$ is the price of a risky asset S, and that the riskless return over time $[0,T]$ is R. Model the future at time $t = T$ using $\Omega = \{\uparrow, \downarrow\}$ and assume that $S(T,\downarrow) < S(T,\uparrow)$.

   (a) Use the no-arbitrage Axiom 1 to conclude that

   $$S(T,\downarrow) < RS(0) < S(T,\uparrow).$$

   (b) Use the Fair Price Theorem 1.4 to prove the same inequalities.

   Solution: (a) First exclude the case $RS(0) \le S(T,\downarrow) < S(T,\uparrow)$, as it offers the following arbitrage opportunity:

   - At time $t = 0$, borrow $S(0)$ from the bank and buy one share of S. Initial cost is 0.
   - At time $t = T$, sell S for $S(t,\omega)$ and repay the loan with interest for $RS(0)$. The net result is $S(T,\omega) - RS(0)$.

   The net at expiry is either $S(T,\downarrow) - RS(0) \ge 0$ or $S(T,\uparrow) - RS(0) > 0$, an arbitrage opportunity as claimed, forbidden by Axiom 1.

   Second, exclude the possibility $S(T,\downarrow) < S(T,\uparrow) \le RS(0)$ as it also provides an arbitrage opportunity:

   - At time $t = 0$, sell S short for $S(0)$ and deposit the money in the bank. Initial cost is again 0.
   - At time $t = T$, withdraw the principal and interest $RS(0)$ from the bank and buy S to cover the short for $S(t,\omega)$. The net result is $RS(0) - S(T,\omega)$.

   The net at expiry in this case is either $RS(0) - S(T,\uparrow) \ge 0$ or $RS(0) - S(T,\downarrow) > 0$ giving an arbitrage opportunity as claimed, forbidden by Axiom 1.

   Conclude that $S(T,\downarrow) < RS(0) < S(T,\uparrow)$.

   (b) From Theorem 1.4 and the definition of expectation,

   $$RS(0) = \Pr(\downarrow)S(T,\downarrow) + \Pr(\uparrow)S(T,\uparrow)$$

But $0 < \Pr(\downarrow) < 1$ since otherwise the future is certain, and thus also $0 < \Pr(\uparrow) < 1$ since $\Pr(\uparrow) = 1 - \Pr(\downarrow)$. Conclude that $RS(0)$ lies strictly inside the interval $[S(T,\downarrow), S(T,\uparrow)]$, as claimed. □

2. In Exercise 1 above, model the future at time $t = T$ using the N-step binomial model $\Omega = \{\omega_0, \omega_1, \ldots, \omega_N\}$ and assume that

$$S(T, \omega_k) = S(0)u^k d^{N-k},$$

where $S(0) > 0$ is the spot price and $0 < d < u$ are the up factor and down factor, respectively, over one time step T/N.

(a) Use the no-arbitrage Axiom 1 to conclude that

$$d < R^{1/N} < u.$$

(b) Use the Fair Price Theorem 1.4 to prove the same inequalities.

Solution: First note that $0 < d < u$ and $S(0) > 0$ together imply that

$$0 < S(T, \omega_0) = S(0)d^N < S(T, \omega_1) < \cdots < S(T, \omega_N) = S(0)u^N$$

(a) If $R^{1/N} \leq d < u$, then $R \leq d^N < u^N$, which implies

$$RS(0) \leq S(T, \omega_0); \qquad RS(0) < S(T, \omega_k), \quad k = 1, \ldots, N,$$

so that borrowing $S(0)$ to buy S at $t = 0$ will result in an arbitrage profit at $t = T$. Similarly, if $d < u \leq R^{1/N}$, then $d^N < u^N \leq R$, which implies

$$RS(0) \geq S(T, \omega_N); \qquad RS(0) > S(T, \omega_k), \quad 0 \leq k < N,$$

so that selling S short for $S(0)$ and investing the money risklessly at $t = 0$ will result in an arbitrage profit at $t = T$.

Both cases are forbidden by Axiom 1, so $d < R^{1/N} < u$.

(b) From Theorem 1.4 and the definition of expectation,

$$RS(0) = \mathrm{E}(S(T)) = \sum_{k=0}^{N} \Pr(\omega_k) S(T, \omega_k), = S(0) \sum_{k=0}^{N} \Pr(\omega_k) u^k d^{N-k}.$$

Dividing by $S(0) > 0$ simplifies this to

$$R = \sum_{k=0}^{N} \Pr(\omega_k) u^k d^{N-k}.$$

Now $0 < d < u$ implies that $d^N < ud^{N-1} < \cdots < u^{N-1}d < u^N$, and the probabilities lie in $[0, 1]$ and sum to 1, so the right-hand side is a convex combination of points in the interval $[d^N, u^N]$. Since the asset is risky, at least two of the states have positive probabilities. The convex combination must therefore lie strictly inside the interval, so

$$d^N < R < u^N,$$

from which it follows that $d < R^{1/N} < u$ as claimed. □

3. Suppose that a portfolio X contains risky stock S and riskless bond B in amounts h_0, h_1:

$$X(t,\omega) = h_0 B(t,\omega) + h_1 S(t,\omega).$$

Model the future at time $t = T$ using $\Omega = \{\uparrow, \downarrow\}$, assuming only that $S(T,\uparrow) \neq S(T,\downarrow)$ and that $B(T,\uparrow) = B(T,\downarrow) = R$. Compute h_0 and h_1 in terms of all the other quantities. (Hint: use Macsyma to derive Eq.3.1.)

Solution: Set up the system of equations at $t = T$:

$$\begin{aligned} X(T,\uparrow) &= h_0 B(T,\uparrow) + h_1 S(T,\uparrow) &= h_0 R + h_1 S(T,\uparrow) \\ X(T,\downarrow) &= h_0 B(T,\downarrow) + h_1 S(T,\downarrow) &= h_0 R + h_1 S(T,\downarrow) \end{aligned}$$

Use these Macsyma commands to solve the system:

```
eq1: xTu=h0*R+h1*sTu;   /* Up state equation */
eq2: xTd=h0*R+h1*sTd;   /* Down state equation */
h0h1: solve([eq1,eq2],[h0,h1]);  /* Solve for h0,h1 */
```

That results in this output:

```
[[h0=-(sTd*xTu-sTu*xTd)/((sTu-sTd)*R),
    h1=(xTu-xTd)/(sTu-sTd)]]
```

Writing the Macsyma solution in the orginal notation gives

$$h_0 = \frac{S(T,\uparrow)X(T,\downarrow) - S(T,\downarrow)X(T,\uparrow)}{(S(T,\uparrow) - S(T,\downarrow))R}; \quad h_1 = \frac{X(T,\uparrow) - X(T,\downarrow)}{S(T,\uparrow) - S(T,\downarrow)}.$$

□

4. In Exercise 3 above, suppose that X is a European-style Call option for S with expiry T and strike price K. Use the payoff formula $X(T) = [S(T) - K]^+$ in the equation for h_1 to prove that

$$0 \leq h_1 \leq 1.$$

Conclude that, in this model of the future, a European-style Call option for S is equivalent to a portfolio containing part of a share of S plus or minus some cash.

Solution: Substitute the payoff formula into the equation for h_1, then add and subtract K in the denominator to get:

$$h_1 = \frac{X(T,\uparrow) - X(T,\downarrow)}{S(T,\uparrow) - S(T,\downarrow)} = \frac{[S(T,\uparrow) - K]^+ - [S(T,\downarrow) - K]^+}{(S(T,\uparrow) - K) - (S(T,\downarrow) - K)}.$$

It may be assumed that $S(T,\uparrow) > S(T,\downarrow)$, since the states can be switched without changing the value of h_1. Then there are three cases to consider:

Case 1: If $S(T,\uparrow) > S(T,\downarrow) > K$, then both plus-parts are positive, so

$$h_1 = \frac{(S(T,\uparrow) - K) - (S(T,\downarrow) - K)}{(S(T,\uparrow) - K) - (S(T,\downarrow) - K)} = 1.$$

Case 2: If $K \geq S(T,\uparrow) > S(T,\downarrow)$, then both plus-parts are zero, so

$$h_1 = \frac{0 - 0}{(S(T,\uparrow) - K) - (S(T,\downarrow) - K)} = 0.$$

Case 3: If $S(T,\uparrow) > K \geq S(T,\downarrow)$, then the first plus-part is positive but the second is zero, so

$$h_1 = \frac{(S(T,\uparrow) - K) - 0}{(S(T,\uparrow) - K) - (S(T,\downarrow) - K)}.$$

But the denominator is $(S(T,\downarrow) - K) \leq 0$, which implies

$$(S(T,\uparrow) - K) - (S(T,\downarrow) - K) \geq (S(T,\uparrow) - K) > 0,$$

so the positive denominator is no smaller than the positive numerator, so $0 < h_1 \leq 1$.

Conclude that $0 \leq h_1 \leq 1$ in all cases. □

5. In Exercise 3 above, suppose that X is a European-style Put option for S with expiry T and strike price K. Use the payoff formula $X(T) = [K - S(T)]^+$ in the equation for h_1 to prove that

$$-1 \leq h_1 \leq 0.$$

Conclude that, in this model of the future, a European-style Put option for S is equivalent to a portfolio containing part of a share of S sold short plus or minus some cash.

Solution: Substitute the payoff formula into the equation for h_1, multiply numerator and denominator by -1, and then add and subtract K in the denominator to get:

$$h_1 = \frac{X(T,\uparrow) - X(T,\downarrow)}{S(T,\uparrow) - S(T,\downarrow)} = -\frac{[K - S(T,\uparrow)]^+ - [K - S(T,\downarrow)]^+}{(K - S(T,\uparrow)) - (K - S(T,\downarrow))}.$$

It may be assumed that $S(T,\uparrow) > S(T,\downarrow)$, since the states can be switched without changing the value of h_1. Then there are three cases to consider:

Case 1: If $K > S(T,\uparrow) > S(T,\downarrow)$, then both plus-parts are positive, so

$$h_1 = -\frac{(K - S(T,\uparrow)) - (K - S(T,\downarrow))}{(K - S(T,\uparrow)) - (K - S(T,\downarrow))} = -1.$$

Case 2: If $S(T,\uparrow) > S(T,\downarrow) \geq K$, then both plus-parts are zero, so

$$h_1 = -\frac{0-0}{(K-S(T,\uparrow))-(K-S(T,\downarrow))} = 0.$$

Case 3: If $S(T,\uparrow) \geq K > S(T,\downarrow)$, then the first plus-part is zero but the second is positive, so

$$\begin{aligned} h_1 &= -\frac{0-(K-S(T,\downarrow))}{(K-S(T,\uparrow))-(K-S(T,\downarrow))} \\ &= \frac{(K-S(T,\downarrow))}{(K-S(T,\uparrow))-(K-S(T,\downarrow))}. \end{aligned}$$

But the denominator is $(K-S(T,\uparrow)) \leq 0$, which implies

$$(K-S(T,\uparrow))-(K-S(T,\downarrow)) \leq -(K-S(T,\downarrow)) < 0,$$

so the negative denominator has no smaller absolute value than the positive numerator, so $-1 \leq h_1 < 0$.

Conclude that $-1 \leq h_1 \leq 0$ in all cases.

Remark. An alternative proof uses the identity $y = [y]^+ - [-y]^+$ which is true for any y. Then

$$(S(T,\omega)-K) = [S(T,\omega)-K]^+ - [K-S(T,\omega)]^+,$$

so

$$[K-S(T,\omega)]^+ = [S(T,\omega)-K]^+ - (S(T,\omega)-K),$$

and thus

$$h_1 = \frac{[S(T,\uparrow)-K]^+ - [S(T,\downarrow)-K)]^+}{(S(T,\uparrow)-K)-(S(T,\downarrow)-K)} - 1.$$

The result now follows from the Call h_1 inequalities. □

6. Suppose that $C(0)$ and $P(0)$ are the premiums for European-style Call and Put options, respectively, on an asset S with the following parameters: expiry at $T = 1$ year, spot price $S(0) = 90$, strike price $K = 95$. Assume that the riskless annual percentage rate is $r = 0.02$, and the volatility for S is $\sigma = 0.15$, and that these will remain constant from now until expiry.

   (a) Use a LibreOffice Calc spreadsheet to implement the Cox-Ross-Rubinstein (CRR) model to compute $C(0)$ and $P(0)$ with $N = 10$ time steps, using the backward pricing formula in Eq.3.18. (Hint: compare output with `CRReurAD()` to check for bugs.)

   (b) Use the Octave function `CRReurAD()` with $N = 10$, $N = 100$, and $N = 1000$ time steps to compute $C(0)$ and $P(0)$.

(c) Repeat part (b) with the Octave function `CRReur()` on p.88, again using $N = 10$, $N = 100$, and $N = 1000$ time steps to compute $C(0)$ and $P(0)$. Profile the time required to compute them, and compare the time and the output with that of `CRReurAD()`.

(d) Compare the prices from parts (b) and (c). Is it justified to use $N = 1000$? Is $N = 10$ sufficiently accurate?

Solution: (a) See the spreadsheet `CRR.ods` in the programs archive. With $N = 10$ time steps, rounding to five significant digits, it computes $C(0) =$ \$4.1733 and $P(0) =$ \$7.2922.

(b) Implement the program `CRReurAD()` on p.76, input the parameters, and compute the CRR approximations at $N = 10$, $N = 100$, and $N = 1000$ with the commands

```
T=1; S0=90; K=95; r=0.02; v=0.15;
[C0,P0]=CRReurAD(T,S0,K,r,v,10) % 4.1733, 7.2922
[C0,P0]=CRReurAD(T,S0,K,r,v,100) % 4.0572, 7.1761
[C0,P0]=CRReurAD(T,S0,K,r,v,1000) % 4.0555, 7.1744
```

At $N = 10$ time steps it returns $C(0) =$ \$4.1733 and $P(0) =$ \$7.2922, in agreement with the spreadsheet.

After $N = 100$ time steps, $C(0) =$ \$4.0572 and $P(0) =$ \$7.1761.

After $N = 1000$ time steps, $C(0) =$ \$4.0555 and $P(0) =$ \$7.1744.

In all cases the computations are almost instantaneous, requiring no noticeable time.

(c) The Octave program `CRReur()m` on p.88 uses the backward induction formula to compute Call and Put premiums. It therefore fills two recombining binomial trees of depth N at a cost of $O(N^2)$ compared with the $O(N)$ cost of the Arrow-Debreu expansion method.

Input the parameters and compute the CRR approximations at $N = 10$, $N = 100$, and $N = 1000$ with the commands

```
T=1; S0=90; K=95; r=0.02; v=0.15;
[C,P]=CRReur(T,S0,K,r,v,10); C(1,1),P(1,1) % 4.1733, 7.2922
profile on; CRReur(T,S0,K,r,v,10); profshow % ~0.013 seconds
[C,P]=CRReur(T,S0,K,r,v,100); C(1,1),P(1,1) % 4.0572, 7.1761
profile on; CRReur(T,S0,K,r,v,100); profshow % ~1.05 seconds
[C,P]=CRReur(T,S0,K,r,v,1000); C(1,1),P(1,1) % 4.0555, 7.1744
profile on; CRReur(T,S0,K,r,v,1000); profshow % ~103 seconds
```

Note that the outputs are pairs of matrices, so that to get just the premiums it is necessary to extract just the $(1, 1)$ element.

At $N = 10$ time steps it almost instantaneously returns $C(0) =$ \$4.1733 and $P(0) =$ \$7.2922, in agreement with the spreadsheet. Profiled time was 0.013 seconds.

With $N = 100$ time steps it very quickly computes $C(0) = \$4.0572$ and $P(0) = \$7.1761$. Profiled time was 1.05 seconds.

With $N = 1000$ time steps it takes a considerably longer time to compute $C(0) = \$4.0555$ and $P(0) = \$7.1744$. Profiled time was 102 seconds.

Note that the ratios of profiled times agree with the $O(N^2)$ complexity estimate.

(d) $N = 100$ seems justified since the prices are quite different from the $N = 10$ values. It seems unjustified to use $N = 1000$, which costs much more time and space (using the backward induction algorithm) but gives almost the same result as $N = 100$. □

7. Compare the prices from parts (a) and (b) of previous Exercise 6 with the Black-Scholes prices computed using Eqs.2.25 and 2.26. Plot the logarithm of the differences against $\log N$ to estimate the rate of convergence. (Hint: Use the programs in Chapter 2, Section 2.4.)

   Solution: Use the parameters from Exercise 6 in the Octave program `BS()` on p.34, as follows:

```
T=1; S0=90; K=95; r=0.02; v=0.15; [C0,P0]=BS(T,S0,K,r,v)
```

   This returns $C_{BS} = \texttt{C0} = 4.0548$ and $P_{BS} = \texttt{P0} = 7.1736$. Now compute the logarithms of the differences as a function of $\log N$:

```
Ns=[10,100,1000]; log(Ns)
C0=4.0548; CCRR=[4.1733,4.0572,4.0555]; log(abs(CCRR-C0))
P0=7.1736; PCRR=[7.2922,7.1761,7.1744]; log(abs(PCRR-P0))
```

   The output is tabulated below:

N	$\log N$	$\log \lvert C_{CRR} - C_{BS} \rvert$	$\log \lvert P_{CRR} - P_{BS} \rvert$
10	2.3026	−2.1328	−2.1320
100	4.6052	−6.0323	−5.9915
1000	6.9078	−7.2644	−7.1309

   Finally, generate the log-log plots:

```
plot(log(Ns),log(abs(CCRR-C0))); title("Call Difference");
xlabel("log N"); ylabel("log|CRR(N)-BS|");   figure;
plot(log(Ns),log(abs(PCRR-P0))); title("Put Difference");
xlabel("log N"); ylabel("log|CRR(N)-BS|");
```

   The results may be seen in Figure 1. For both Call and Put differences, the graphs are close to lines of slope −1, suggesting that the difference between Black-Scholes and its N-step CRR approximation is $O(N^{-1})$. This may be quantified by regression using `polyfit(x,y,1)`:

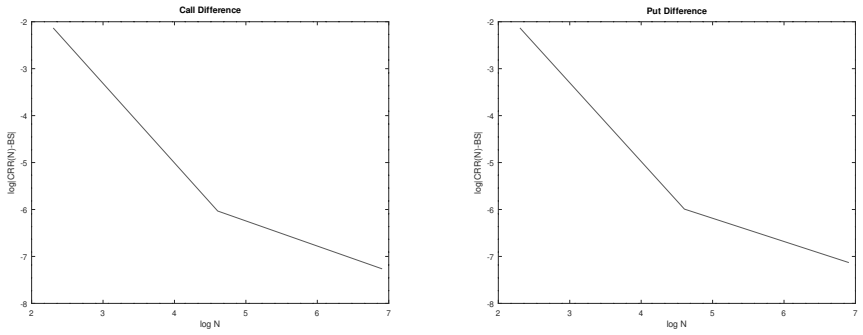


Figure 1: (From Exercise 7) Log-log plots showing the differences between Black-Scholes prices and their N-step CRR approximations, for certain European-style Call and Put options, as a function of N.

```
polyfit(log(Ns),log(abs(CCRR-C0)),1) % -1.114310  -0.011598
polyfit(log(Ns),log(abs(PCRR-P0)),1) % -1.085497  -0.085887
```

The first output number is the slope of the least-squares line fitting the data, in both cases close to -1. The second is the intercept; it is an estimate for the logarithm of the constant in the $O(N^{-1})$ rate.

Remark. Using only 5 significant digits introduces substantial round-off error at large N, where the differences are small. This is unavoidable since the parameters are only specified to 2 or 3 significant digits. □

8. Derive 3.32 on p.79:

$$q = \frac{1}{2} + \frac{r + \frac{\sigma^2}{2}}{2\sigma}\sqrt{\frac{T}{N}} + O\left(\frac{T}{N}\right).$$

Solution: Recall that $q = (u - 1/R)/(u - 1/u)$. Using Taylor's approximation in the numerator gives

$$[1 + \sigma\sqrt{\frac{T}{N}} + \frac{\sigma^2}{2}\frac{T}{N} + O(\sqrt{\frac{T}{N}}^3)] - [1 - \frac{rT}{N} + O([\frac{T}{N}]^2)],$$

while in the denominator it gives

$$[1+\sigma\sqrt{\frac{T}{N}}+\frac{\sigma^2}{2}\frac{T}{N} + O(\sqrt{\frac{T}{N}}^3)] - [1-\sigma\sqrt{\frac{T}{N}}+\frac{\sigma^2}{2}\frac{T}{N} + O(\sqrt{\frac{T}{N}}^3)].$$

Canceling terms and simplifying the ratio gives

$$\begin{aligned} q &= \frac{\sigma\sqrt{\frac{T}{N}} + \left(r + \frac{\sigma^2}{2}\right)\frac{T}{N} + O\left(\sqrt{\frac{T}{N}}^3\right) + O\left(\left[\frac{T}{N}\right]^2\right)}{2\sigma\sqrt{\frac{T}{N}} + O\left(\sqrt{\frac{T}{N}}^3\right)} \\ &= \frac{1}{2} + \frac{r + \frac{\sigma^2}{2}}{2\sigma}\sqrt{\frac{T}{N}} + O\left(\frac{T}{N}\right), \end{aligned}$$

as claimed. □

9. Use the CRR approximation with $N = 4$ to compute the European-style Call option premiums at several hundred equally spaced spot prices $75 \leq S_0 \leq 115$, with expiry $T = 1$, strike $K = 95$, $r = 0.02$, and $\sigma = 0.15$.

   (a) Plot the values against S_0.

   (b) At what values of S_0 in that range does the graph appear to be nonsmooth?

   (c) Compute the points of nondifferentiability for S_0 in $[75, 115]$.

   Solution: (a) Use `CRReurAD()` in the following Octave code:

```
T=1; K=95; r=0.02; v=0.15; N=4; m=401;
S=linspace(75,115,m); CS=zeros(size(S));
for i=1:m
  S0=S(i); [C0,P0]=CRReurAD(T,S(i),K,r,v,N);  CS(i)=C0;
end
plot(S,CS); title("CRR with N=4");xlabel("S0");ylabel("C0");
```

   See the result in Figure 2.

   (b) The graph appears piecewise linear with joints $\hat{S}_0 \in \{82, 95, 110\}$ where the Call premium is not differentiable with respect to S_0.

   (c) Compute the joints, or points of nondifferentiability $\hat{S}_0$ nearest K, using Eq.3.39 and $j \in \{N/2 - 1, N/2, N/2 + 1\} = \{1, 2, 3\}$:

$$\hat{S}_0 \in \left\{ \frac{K}{u^{2(j+1)-N}}, \frac{K}{u^{2j-N}}, \frac{K}{u^{2(j-1)-N}} \right\} = \left\{ d^2K, K, u^2K \right\},$$

   where $1/d = u = \exp(\sigma\sqrt{T/N})$. With the given parameters,

$$u = \exp(0.15\sqrt{1/4}) = 1.0779, \implies \hat{S}_0 \in \{81.767,\ 95,\ 110.37\},$$

   in good agreement with the visual estimate. □

10. Compute the CRR option premiums and Greeks for European-style Call and Put options on a risky asset with the following parameters: spot price \$90, strike price \$95, expiry in 1 year, annual riskless rate 2%, and volatility 15%. Use $N = 100$ steps. Justify the method used.

    Solution: First compute the option premiums with `CRReurAD()`:

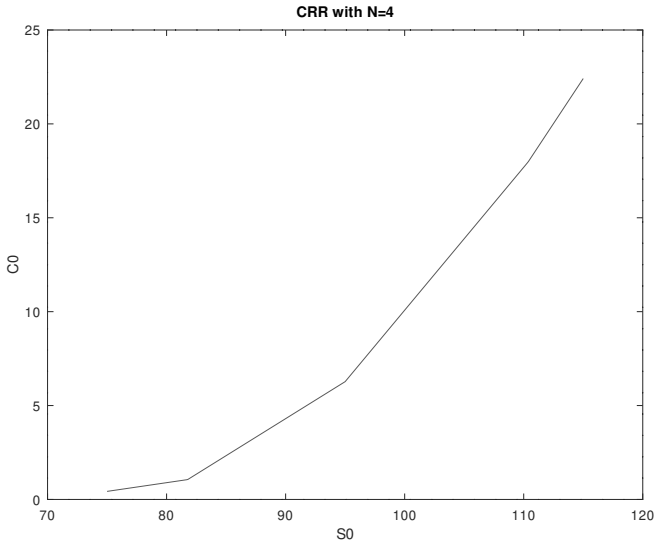


Figure 2: (From Exercise 9) CRR approximation with $N = 4$ to the European-style Call option premium $C(0)$, as a function of spot price $S(0)$.

```
T=1; S0=90; K=95; r=0.02; v=0.15; N=100;
[C0,P0]=CRReurAD(T,S0,K,r,v,N)
```

For Δ and Γ, use the interpolation method on p.83:

```
h0=2*S0*v*sqrt(T/N);  % critical h
u2=exp(2*v*sqrt(T/N)); % squared up factor
x=[S0/u2, S0, S0*u2]-S0; % shifted abscissas
[C0,P0]=CRReurAD(T,S0,K,r,v,N);
[C0u,P0u]=CRReurAD(T,S0*u2,K,r,v,N); % ...at S0*u^2
[C0d,P0d]=CRReurAD(T,S0/u2,K,r,v,N); % ...at S0/u^2
yC=[C0d, C0, C0u]; % Call ordinates
p=polyfit(x,yC,2); DeltaC=p(2), GammaC=2*p(1)
yP=[P0d, P0, P0u]; % Put ordinates
p=polyfit(x,yP,2); DeltaP=p(2), GammaP=2*p(1)
```

This is necessary because the approximation is not differentiable with respect to S_0.

For the other Greeks, use the centered difference approximation to the derivative with h set to 10% of the abscissa value:

```
h=0.10*T; % for ThetaC, ThetaP
[C0u,P0u]=CRReurAD(T+h,S0,K,r,v,N);
[C0d,P0d]=CRReurAD(T-h,S0,K,r,v,N);
```

```
ThetaC=-(C0u-C0d)/(2*h), ThetaP=-(P0u-P0d)/(2*h)
h=0.10*v; % for VegaC, VegaP
[C0u,P0u]=CRReurAD(T,S0,K,r,v+h,N);
[C0d,P0d]=CRReurAD(T,S0,K,r,v-h,N);
VegaC=(C0u-C0d)/(2*h), VegaP=(P0u-P0d)/(2*h)
h=0.10*r; % for RhoC, RhoP
[C0u,P0u]=CRReurAD(T,S0,K,r+h,v,N);
[C0d,P0d]=CRReurAD(T,S0,K,r-h,v,N);
RhoC=(C0u-C0d)/(2*h), RhoP=(P0u-P0d)/(2*h)
```

Since the centered difference approximation has $O(h^2) \approx 1\%$ relative error, expect roughly two significant digits of accuracy. Much smaller values of h are not justified since the option premiums are only $O(1/N) \approx 1\%$ accurate as shown in Exercise 7.

The results are tabulated below:

	Option	Delta	Gamma	Theta	Vega	Rho
Call	4.0572	0.43987	0.029063	−3.4348	36.298	35.457
Put	7.1761	−0.56013	0.029063	−1.5724	36.298	−57.661

Comparison with the Black-Scholes premiums and Greeks computed in Chapter 2, Exercise 10 shows good agreement. □