Ma 405: Introduction to Computational Mathematics
Reading and Homework Assignment 7

Prof. Wickerhauser

You are allowed to collaborate with one other person on homework, but then you must indicate both contributor's names on your solution sheet.

You are also encouraged to work additional exercises from the indicated sections, although the homework grade will be based only on the exercises listed below.

Please return your solutions to me by the end of class on the due date. **Late homework will not be accepted.**

Read Chapter 4, sections 4.3 and 4.4

Pp.120–121, Exercises 4.3.

4.3.7, 4.3.8 ("ith row" should be "ith column"), 4.3.10

Pp.132 Exercises 4.4.

4.4.2

Extra problem. Let H be the $n \times n$ Hilbert matrix defined by

$$H = \begin{pmatrix} 1 & 1/2 & 1/3 & \dots & 1/n \\ 1/2 & 1/3 & & & 1/(n+1) \\ 1/3 & & \ddots & & \vdots \\ \vdots & & & & \\ 1/n & & \dots & & 1/(2n-1) \end{pmatrix}.$$

Let b be the vector $(1, 1, \dots, 1)$ of n components. (a) Use your Gaussian elimination computer program with double precision arithmetic to solve the system $Hx = b$ for $n = 2$, $n = 4$ and $n = 8$, then compute $\|Hx - b\|/\|b\|$. (b) Estimate $\mathrm{cond}(H)$ as a function of n as $n \to \infty$, using any matrix norm you like.