QUIZ 4

Show your work not just your final answer

(1) Let f, g, h be functions from $\mathbb{R}$ to itself which are linearly independent in the vector space of all such functions from reals to itself. Define elements of the vector space of signals by $F(k) = f(k), G(k) = g(k), H(k) = h(k), k \in \mathbb{Z}$. Are F, G, H linearly independent?
(2) Show that if P is a stochastic matrix, then so is P^2.
(3) Decide which of the following are true (with justifications always).
   (a) If A is a square matrix and λ, μ are two eigenvalues of A then $\lambda\mu$ is an eigenvalue of A^2.
   (b) An eigenvalue of a matrix A is also an eigenvalue of A^T.
   (c) An $n \times n$ matrix with n linearly independent eigenvectors is invertible.
(4) Let $p(t) = c_0 + c_1 t + \cdots + c_n t^n$ be a polynomial and A a square matrix. Define $p(A)$ to be $c_0 I + c_1 A + \cdots + c_n A^n$. Show that if λ is an eigenvalue of A, then $p(\lambda)$ is an eigenvalue of $p(A)$.
(5) Let

$$A = \begin{bmatrix} 0.4 & -0.3 \\ 0.4 & 1.2 \end{bmatrix}$$

Calculate $\lim_{k\to\infty} A^k$.
(6) If A is an $n \times n$ real matrix such that $A = A^T$ (such matrices are called *symmetric*), show that its eigenvalues are real.
(7) Find an invertible matrix P with main diagonal elements $2, 1$, and a matrix C of the form

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

such that

$$PCP^{-1} = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

(8) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ is given as $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 1 & 5 & 7 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

Find an ordered basis $\mathcal{B}$ of $\mathbb{R}^3$, with all coordinates being integers, so that the matrix of T relative to $\mathcal{B}$ (for both $\mathbb{R}^3$) is diagonal.

(9) Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Show that the characteristic polynomial is $p(t) = \lambda^2 - (a + d)\lambda + \det A$. Then show that $p(A) = A^2 - (a+b)A + \det AI = 0$.

(10) Find all real eigenvalues and the corresponding eigenvectors for

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

(11) Find the steady-state vector for

$$\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}$$

(12) Decide which of the following are true (with justifications)

(a) If A, B are two $n \times n$ matrices and λ is an eigenvalue for both, then it is an eigenvalue of $A + B$.

(b) If A, B are two $n \times n$ matrices and $\mathbf{x}$ is an eigenvector for both, then $\mathbf{x}$ is also an eigenvector for $A + B$.

(c) If A is an $n \times n$ matrix, then $A - I_n$ is invertible if 1 is not an eigenvalue of A.