QUIZ 3

Show your work not just your final answer

(1) Compute $\det(A^5)$ for

$$A = \begin{pmatrix} 1 & 5 & 7 \\ 0 & 1 & -2 \\ 0 & -1 & 4 \end{pmatrix}.$$

(2) Suppose that B is a 4×4 matrix with $\det B = 3$. Let $C = 2B^T$. Compute $\det C$.

(3) Find the matrix of cofactors of the following matrix:

$$\begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

(4) Find the area of the parallelogram with vertices $(0,0)$, $(3,3)$, $(8,4)$, and $(5,1)$.

(5) Let V be the subset of the 4×4 matrices consisting of those with zeros at every main diagonal element. Is V a subspace?

(6) Which of these subsets of P_9, the polynomials $p(t)$ with degree 9 or less, is a subspace:
   (a) $\{p(t) : p(0) = 17\}$
   (b) $\{p(t) : \int_0^1 p(t)\,dt = 17\}$
   (c) $\{p(t) : p'(0) = p(17)\}$

(7) Let

$$A = \begin{bmatrix} 2 & -2 & -1 \\ 8 & -9 & -6 \\ 10 & -7 & 5 \\ 2 & 2 & 3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ k \end{bmatrix}$$

Find k such that $\mathbf{b}$ is in the column space of A.

(8) Let $T : \mathbf{R}^5 \to \mathbf{R}^3$ be defined by

$$T(x_1, x_2, x_3, x_4, x_5) = \begin{bmatrix} x_1 + 4x_4 - 2x_5 \\ 3x_1 + x_2 - 9x_5 \\ 5x_1 - x_2 + x_3 \end{bmatrix}.$$

Find the dimension of the range of T and the dimension of the kernel of T.

(9) Suppose that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a set of linearly independent vectors. Which of the following sets of vectors is also linearly independent?
   (a) $\{\mathbf{0}, \mathbf{v}, \mathbf{w}\}$
   (b) $\{\mathbf{v}, \mathbf{w}\}$
   (c) $\{\mathbf{u}, \mathbf{v}, \mathbf{u} + \mathbf{v}\}$
   (d) $\{\mathbf{w}\}$
   (e) $\{\mathbf{u}, \mathbf{v} + \mathbf{w}\}$
   (f) $\{\mathbf{u}, \mathbf{u} + \mathbf{v}, \mathbf{u} + \mathbf{v} + \mathbf{w}\}$

(10) Let $S = \{1, 1 + t, 1 + t + t^2\}$ be a subset of $\mathbf{P}_2$, the polynomials of degree 2 or less.
   (a) Is S a linearly independent set?
   (b) Is S a basis for $\mathbf{P}_2$?

(11) Consider the ordered basis B of $\mathbf{R}^2$ consisting of the vectors

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Find $\mathbf{x} \in \mathbf{R}^2$ whose coordinates with respect to this basis are

$$[\mathbf{x}]_B = \begin{bmatrix} 3 \\ 7 \end{bmatrix},$$

(12) Consider the following ordered basis B for the vector space of 2×2 lower triangular matrices:

$$B = \left\{ \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right\}$$

Find the coordinates $[M]_B$ of

$$M = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$$

with respect to this basis.