QUIZ 1

Show your work not just your final answer

(1) Solve the system of linear equations.

$$\begin{aligned} x + y &= 6 \\ 5x - 4y &= 39 \\ 16x - 11y &= 123 \end{aligned}$$

(2) Consider a linear system whose augmented matrix is,

$$\begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 2 & -5 & 1 \\ 6 & 15 & k & 1 \end{bmatrix}$$

Find k so that the system is inconsistent.

(3) How many pivots does the following matrix have?

$$\begin{bmatrix} 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

(4) Decide whether the augmented matrix given in echelon form below is consistent (that is, the corresponding system of linear equations is consistent) and if so, does it have a unique solution.

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

(5) Let $\mathbf{x}, \mathbf{y}$ be vectors and let $\mathbf{z} = p\mathbf{x} + q\mathbf{y}$, where p, q are real numbers. For what values of p, q must Span$\{\mathbf{x}, \mathbf{z}\}$be equal to Span$\{\mathbf{y}, \mathbf{z}\}$?

(6) Let $A = \begin{bmatrix} -8 & 3 \\ -8 & -1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -39 \\ -19 \end{bmatrix}$. Solve for $\mathbf{x}$, given $A\mathbf{x} = \mathbf{b}$.

(7) Solve the equation $-8x - 2y + 9z = 0$ in parametric form. In other words, find two solutions $\mathbf{u}, \mathbf{v}$, 3-vectors, so that all solutions are of the form $s\mathbf{u} + t\mathbf{v}$ for some constants s, t.

(8) Consider the following system of linear equations.

$$\begin{array}{rrrrcl} 2w & -x & +y & -z & = & 4 \\ w & +x & -y & +z & = & 4 \\ 3w & & & & = & 8 \\ 3w & -3x & +3y & -3z & = & 4 \end{array}$$

Write solutions to these as w, x in terms of y, z.

(9) Let $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 6 \\ 3 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ -11 \\ 6 \end{bmatrix}$. Decide whether these are linearly independent.

(10) Let

$$A = \begin{bmatrix} -1 & -3 & 1 \\ 1 & 2 & 0 \\ -1 & 2 & -4 \end{bmatrix}$$

Decide whether $A\mathbf{x} = \mathbf{0}$ has a non-trivial (non-zero) solution.

(11) Let $A = \begin{bmatrix} -4 & -4 & 5 \\ 5 & -6 & -4 \\ 6 & -5 & -4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 21 \\ -2 \\ -11 \end{bmatrix}$. Define a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$. Find a vector $\mathbf{x}$ such that $T(\mathbf{x}) = \mathbf{b}$.

(12) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation, $T(\mathbf{x}) = \begin{bmatrix} x_1 - x_2 \\ x_2 - x_3 \\ x_3 - x_1 \end{bmatrix}$.

Is this onto?