MATH 416 HW 1 SOLUTIONS

p. 7

1. $z+w=(13+5i)+(2-6i)=15-1i$

$w-s=(2-6i)-(1+9i)=1-15i$

$z\cdot s=(13+5i)\cdot(1+9i)=(13-45)+i(5+117)$
$=-32+i\cdot 122.$

$w\cdot s=(2-6i)\cdot(1+9i)=(2+54)+i(18-6)$
$=56+i(12)$

$s-z=(1+9i)-(13+5i)=-12+4i$

3. $z+\bar{z}=2\operatorname{Re} z=12$

$z+2\bar{z}=(6-2i)+(12+4i)=18+2i$

$z-\bar{w}=(6-2i)-(4-3i)=2+i$

$z\cdot\bar{s}=(6-2i)\cdot(-5-i)=(-30-2)+i(-6+10)=-32+4i$

$w\cdot\bar{s}^2=(4+3i)\cdot(-5-i)^2=(4+3i)\cdot(25-1+10i)=(4+3i)(24+10i)$
$=((96-30)+i(72+40))=66+112i$

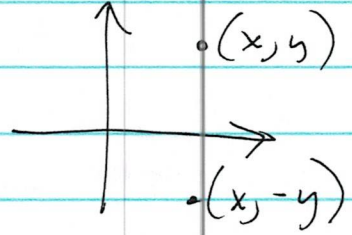
4. Let $z=x+iy$, Then $\bar{z}=x-iy$.

Dist of z to 0 is $|z|=\sqrt{x^2+y^2}$

Dist of $\bar{z}$ to 0 is $|\bar{z}|=\sqrt{x^2+(-y)^2}=\sqrt{x^2+y^2}$

5. $z=x+iy\leftrightarrow(x,y)$

$\bar{z}=(x-iy)\leftrightarrow(x,-y)$.



7, Let $a, b \in \mathbb{R}$. Then $a \leftrightarrow a + i0$

$b \leftrightarrow b + i0$.

$\mathbb{R} \to a+b \leftrightarrow (a+b) + i0 \leftrightarrow (a + i0) + (b + i0) \in \mathbb{C}$

p. 15

1. $|z| = |6 - 9i| = \sqrt{6^2 + (-9)^2} = \sqrt{36 + 81} = \sqrt{117}$

$|w| = |4 + 2i| = \sqrt{4^2 + 2^2} = \sqrt{20}$

$|z + w| = |(6 - 9i) + (4 + 2i)| = |10 - 7i| = \sqrt{10^2 + (-7)^2}$

$= \sqrt{149}$.

$|\beta - w| = |(1 + 10i) - (4 + 2i)| = |-3 + 8i| = \sqrt{(-3)^2 + 8^2} = \sqrt{73}$

$|z - w| = |(6 - 9i) - (4 + 2i)| = |2 - 11i| = \sqrt{2^2 + (-11)^2} = \sqrt{125}$

$|z + w| = |(6 - 9i) + (4 + 2i)| = |10 - 7i| = \sqrt{10^2 + (-7)^2} = \sqrt{149}$

$|\beta - z| = |(1 + 10i) - (6 - 9i)| = |-5 + 19i| = \sqrt{25 + 361} = \sqrt{386}$

$|z + w| \leq |z| + |w|$

$\Leftrightarrow \sqrt{149} \leq \sqrt{117} + \sqrt{20}$

$149 \leq 117 + 20 + 2\sqrt{117 \cdot 20}$

$149 \leq 137 + 2\sqrt{2340}$ TRUE

$|z \cdot w| = |(6 - 9i) \cdot (4 + 2i)| = |(24 + 18) + (-24)i| = \sqrt{42^2 + 24^2} = \sqrt{2340}$

$|z| \cdot |w| = \sqrt{117}\sqrt{20} = \sqrt{2340}$

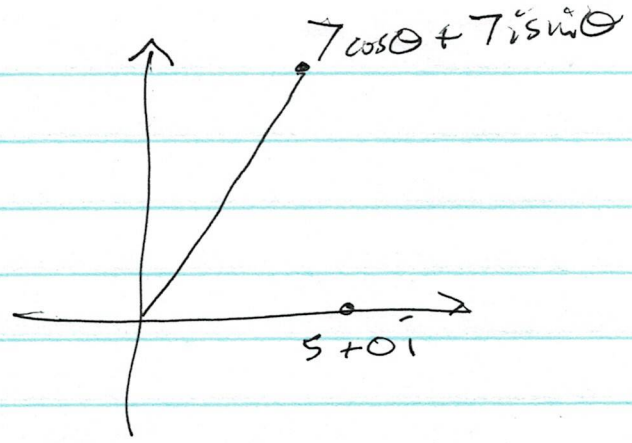
$|s \cdot z| = |(1+10i)\cdot(6-9i)| = |96+51i| = \sqrt{96^2+51^2} = \sqrt{11827}$

$|s| = \sqrt{1^2+10^2} = \sqrt{101}$

$|z| = \sqrt{6^2+9^2} = \sqrt{117}$

$|s|\cdot|z| = \sqrt{101}\cdot\sqrt{117} = \sqrt{11817}$.

2.



We want

$$|7\cos\theta + 7i\sin\theta + 5 + 0i| = 9$$

$$(7\cos\theta + 5)^2 + (7\sin\theta)^2 = 81$$

$$49\cos^2\theta + 70\cos\theta + 25 + 49\sin^2\theta = 81$$

$$49 + 25 + 70\cos\theta = 81$$

$$70\cos\theta = 7$$

$$\cos\theta = \frac{1}{10}$$

$$\sin\theta = \sqrt{1 - \left(\frac{1}{10}\right)^2} = \frac{\sqrt{99}}{10}.$$

So $z = 5 + 0i$

$$w = \frac{1}{10} + \frac{7\sqrt{99}}{10}$$

$$z + w = \frac{57}{10} + \frac{7\sqrt{99}i}{10}$$

It is easy to check that $|z + w| = 9$.

There are actually infinitely many solutions to this problem. Any rotation of the above solution is another.

4. $\frac{1}{z} = \frac{4+6i}{4^2+6^2} = \frac{4+6i}{52} = \frac{2+3i}{26}$

$\frac{1}{w} = \frac{2-7i}{2^2+7^2} = \frac{2-7i}{4+49} = \frac{2-7i}{53}$

$\frac{z}{w} = (4-6i)\cdot\frac{2-7i}{53} = \frac{-34-40i}{53}$

$\frac{w}{z} = (2+7i)\cdot\frac{2+3i}{26} = \frac{-17+20i}{26}$

6 a) closed because contains its boundary. Not open

b) open because strict inequality

c) Neither open nor closed since one equality strict & one not.

d) Open since strict inequality

e) closed because non-strict inequality.

8 Clearly $z=1$ is a root. So the polynomial factors as

$$p(z) = (z-1)(z^2+z-2)$$

The roots of z^2+z-2 are 1 and -2. So 1 is a repeated root of p.

pp. 20-21

1. $e^{\pi i} = \cos\pi + i\sin\pi = -1 + 0i$

$e^{(\pi/3)i} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$5e^{-i\frac{\pi}{4}} = 5\left(\cos-\frac{\pi}{4} + i\sin-\frac{\pi}{4}\right) = 5\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

$2e^{i} = 2(\cos 1 + i\sin 1)$

$7e^{-3i} = 7(\cos -3 + i\sin -3) = 7(\cos 3 - i\sin 3)$.

2. $2 + 2i = \sqrt{8}\,e^{i\frac{\pi}{4}}$

$1 + \sqrt{3}i = 2e^{i\frac{\pi}{3}}$

$\sqrt{3} - i = 2e^{-i\pi/6}$

$\sqrt{2} - \sqrt{2}i = 2e^{-i\pi/4}$

$i = e^{i\frac{\pi}{2}}$

$-1 - i = \sqrt{2}\,e^{i\frac{5\pi}{4}}$

4. $w^5 = z \Rightarrow |w|^5 = |z| = 3 \Rightarrow |w| = 3^{1/5}$.

6. $w^2 = z^3$. If $w = re^{i\theta}$ and $z = se^{i\psi}$ then

$r^2 e^{i2\theta} = s^3 e^{3i\psi}$.

So $r^2 = s^3$, $2\theta = 3\psi$ (up to periodicity).

$\Rightarrow r = s^{3/2}$, $\theta = \frac{3}{2}\psi$.