Math 4111 Fall, 2020
Krantz

FIRST MIDTERM EXAM

General Instructions: Read the statement of each problem carefully. If you want full credit on a problem then you must show your work. If you only write the answer then you will *not* receive full credit.

Be sure to ask questions if anything is unclear. This exam has 8 questions and is worth 100 points. You will have 50 minutes to take this exam.

(8 points) **1.** Prove that the sequence

$$a_j = \frac{(-1)^j \cdot 2}{j+1}$$

converges to 0.

(8 points) **2.** What is the multiplicative inverse of the complex number $2 + 3i$?

(8 points) **3.** Find all cube roots of the complex number $-1 + i$.

(8 points) **4.** What is the least upper bound of the set $\{x \in \mathbb{R} : x^2 < 7\}$? Give a reason for your answer.

(8 points) **5.** Prove that if

$$\liminf_{j \to +\infty} a_j = \limsup_{j \to +\infty} a_j \,,$$

then the sequence $\{a_j\}$ converges.

(9 points) **6.** Discuss convergence and divergence for each of these series.

(a) $\sum_{j=1}^{\infty} \frac{j!}{j^j}$

(b) $\sum_{j=1}^{\infty} \frac{(2j)!}{(3j)!}$

(c) $\sum_{j=2}^{\infty} \frac{1}{j \cdot \log^2 j}$

(9 points) **7.** Let $\sum_j a_j$ and $\sum_j b_j$ be series of positive terms. Prove that, if there is a constant $C > 0$ such that

$$\frac{1}{C} \leq \frac{a_j}{b_j} \leq C$$

for all j large, then either both series converge or both series diverge.

(9 points) **8.** Discuss convergence or divergence of the series

$$\sum_{j=1}^{\infty} \frac{\cos j}{j} .$$

(8 points) **9.** Let $\sum_j a_j$ and $\sum_j b_j$ be convergent series of positive real numbers. Discuss the convergence of $\sum_j (a_j \cdot b_j)$.

(9 points) **10.** If $1/2 > b_j > 0$ and if $\sum_j b_j$ converges then prove that

$$\sum_{j=1}^{\infty} \frac{b_j}{1 - b_j}$$

converges.

(8 points) **11.** Let $\gamma > 0$ be a fixed real number. Give an example of a sequence whose limsup and liminf differ by γ.

(8 points) **12.** Consider $\{a_j\}$ both as a sequence and as a set. How are the limsup of the sequence and the sup of the set related? How are the liminf of the sequence and the inf of the set related?