Math 4111 Fall, 2020
Krantz

PRACTICE EXAM FOR FINAL EXAM

(6 points) **1.** Prove that multiplication is commutative in the complex numbers.

(6 points) **2.** Let S be a set of real numbers. Prove that $\sup S$ and $\inf S$ are unique.

(9 points) **3.** Prove that the series

$$\sum_{j=1}^{\infty} \frac{\cos^2 j}{j}$$

diverges.

(9 points) **4.** Discuss convergence or divergence of the sequence

$$\left\{\frac{j!}{2^j}\right\}_{j=1}^{\infty}.$$

(6 points) **5.** Discuss convergence or divergence of the series

$$\sum_{j=3}^{\infty} \frac{1}{j \log j}.$$

(6 points) **6.** Let each a_j be positive and assume that $\sum_j a_j$ diverges. Prove that $\sum_j a_j/j^{1/2}$ may or may not converge.

(8 points) **7.** Let f be a function that is twice continuously differentiable on the interval $(0, \infty)$. Assume that

$$0 < f''(x) \leq x$$

for all $x \in (0, \infty)$. What can you say about the growth of f at infinity?

(6 points) **8.** Let f be a twice continuously differentiable function on the real line. Assume that $f''(x) \leq -c < 0$ for all x and for some positive constant c. Prove that f is unbounded below.

(8 points) **9.** The intersection of infinitely many open sets need not be open. It need not be closed either. Give examples to illustrate the possibilities.

(8 points) **10.** Let $\{p_j\}_{j=1}^{\infty}$ be a collection of polynomials. Assume that $\sum_j p_j(x) = f(x)$ for every x in the interval $[0, 1]$ for some function f. Prove that f need not be a polynomial.

(8 points) **11.** Prove that the uniform limit of continuously differentiable functions need not be continuously differentiable.

(7 points) **12.** Let $\alpha(x)$ be the greatest integer function: The value of $\alpha(x)$ is the greatest integer less than or equal to x. Calculate

$$\int_0^5 \cos x \, d\alpha(x) \, .$$

(7 points) **13.** Let $f(x) = x^2$ and $g(x) = \sin x$. Calculate

$$\int_0^1 f(x) \, dg(x) \, .$$

(6 points) **14.** Prove that the series

$$\sum_{j=1}^{\infty} \frac{\cos jx}{j^3}$$

converges uniformly to a continuous function on the interval $[0, 1]$.