Math 310 Fall, 2020
Krantz

PRACTICE EXAM FOR FIRST MIDTERM

(8 points) **1.** Write the converse and the contrapositive of the sentence

If down is up, then life goes on.

Label each one.

Converse:

If life goes on, then down is up.

Contrapositive:

If life does not go on, then down is not up.

(8 points) **2.** Are the statements $\mathbf{A} \vee \mathbf{B}$ and $\sim \mathbf{A} \wedge \mathbf{B}$ logically equivalent? Why or why not?

A	B	~A	A ∨ B	~A ∧ B
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

The two columns are different so statements are <u>not</u> logically equivalent.

(10 points) **3.** Express the statement $\forall x, \sim \mathbf{P}(x)$ using $\exists$ instead of $\forall$.

$\sim \exists x, P(x)$

(8 points) **4.** Prove that the square of an odd integer is odd.

Let $m = 2k+1$ be odd. Then

$m^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k) + 1$

is odd.

(10 points) **5.** Prove that the integer 6 does not have a rational square root.

Suppose to the contrary that 6 has a rational square root m/n in lowest terms. So $(m/n)^2 = 6$ hence $m^2 = 6n^2$. So 6 divides the right sidence hence 6 divides the left side. So 2 must divide m and 3 must divide m. Write $m = 3k$. Thus $(3k)^2 = 6n^2 \Rightarrow 9k^2 = 6n^2 \Rightarrow 3k^2 = 2n^2$.

So 3 divides the left side hence 3 divides the right side. We conclude that 3 divides n.

We have shown that 3 divides m and 3 divides n. So (m/n) is <u>not</u> in lowest terms. Contradicts

Hence the rational square root m/n does not exist.

(8 points) **6.** Use mathematical induction to prove that the sum of the first n odd integers is n^2.

a) $P(n)$ is $1+3+\cdots+(2n-1)=n^2$.

b) $P(1)$ is $1=2\cdot 1-1$ which is true.

c) Assume $P(n)$: $1+3+\cdots+(2n-1)=n^2$.

Add $2n+1$ to both sides.

$$1+3+\cdots+(2n-1)+(2n+1)=n^2+2n+1$$

$$1+3+\cdots+(2n+1)=(n+1)^2.$$

This is $P(n+1)$.

The induction is complete.

(8 points) **7.** Give a truth table for the statement $(\mathbf{A}\vee\sim\mathbf{B})\Rightarrow(\sim\mathbf{A}\wedge\mathbf{B})$.

A	B	~A	~B	A∨~B	~A∧B	(A∨~B) ⇒ (~A∧B)
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	F	F

(8 points) **8.** Use any method to prove that $2^k > 1 + 2k$ for $k > 2$.

By induction.

a) $P(k)$ is $2^k > 1+2k$

b) $P(3)$ is $2^3 > 1+2\cdot 3$ which is true.

c) Assume $P(k)$: $2^k > 1 + 2k$.

Multiply both sides by 2:

$$2^{k+1} > 2 + 4k$$
$$\geq 2 + (1+2k)$$
$$= 1 + 2(k+1).$$

This is $P(k+1)$. The induction is complete.

(8 points) **9.** Let S, T, and U be sets. Prove that

$$S \setminus (T \cap U) = (S \setminus T) \cup (S \setminus U).$$

Let $x \in S \setminus (T \cap U)$. Then $x \in S$ and $x \notin T \cap U$.

So $x \in S$ and either $x \notin T$ or $x \notin U$. Thus

$x \in S \setminus T$ or $x \in S \setminus U$. In conclusion, $x \in (S \setminus T) \cup (S \setminus U)$.

So $S \setminus (T \cap U) \subseteq (S \setminus T) \cup (S \setminus U)$.

Now let $x \in (S \setminus T) \cup (S \setminus U)$. Then

x is in S but not T or x is in S but not U.

Hence $x \in S$ but $x \notin T \cap U$. So $x \in S \setminus (T \cap U)$.

Thus $(S \setminus T) \cup (S \setminus U) \subseteq S \setminus (T \cap U)$.

Combining the two inclusions gives

$$S \setminus (T \cap U) = (S \setminus T) \cup (S \setminus U).$$

(9 points) **10.** Let $S = \{a, b, c, d, e\}$, $T = \{b, d, f, h\}$, and $U = \{a, d, g\}$. Calculate

(a) $(S \setminus T) \cup U$

(b) $(S \cap T) \setminus U$

(c) $(S \cup T) \cap U$

a) $S \setminus T = \{a, c, e\}$

$(S \setminus T) \cup U = \{a, c, d, e, g\}$

b) $S \cap T = \{b, d\}$

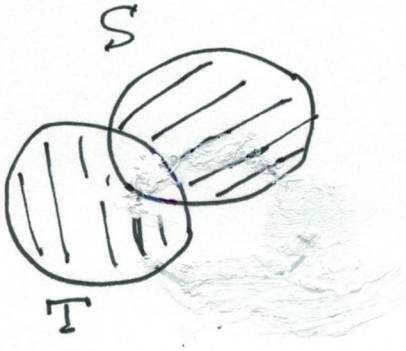
$(S \cap T) \setminus U = \{b\}$

c) $S \cup T = \{a, b, c, d, e, f, h\}$

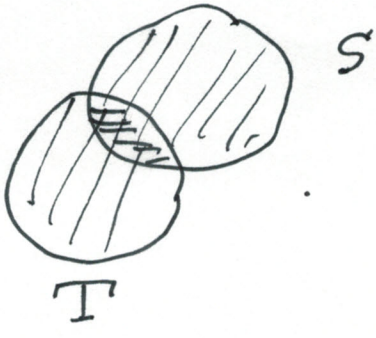
$(S \cup T) \cap U = \{a, d\}$

(8 points) **11.** Let S, T, and U be sets. Draw two Venn diagrams to illustrate the identity

$$(S \setminus T) \cup (T \setminus S) = (S \cup T) \setminus (S \cap T).$$



$(S \setminus T) \cup (T \setminus S)$



$(S \cup T) \setminus (S \cap T)$

(7 points) **12.** Calculate the power set of $\{1, @, *, \gamma\}$.

$\{\{1\}, \{@\}, \{*\}, \{\gamma\}, \{1, @\}, \{1, *\}, \{1, \gamma\},$
$\{@, *\}, \{@, \gamma\}, \{*, \gamma\}, \{1, @, *\}, \{1, @, \gamma\},$
$\{1, *, \gamma\}, \{@, *, \gamma\}, \{1, @, *, \gamma\}, \emptyset\}$