MATH 309

SOLUTIONS TO FIRST PRACTICE MIDTERM

1. $\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{array}\right]$

$x_3 = 1$

$x_2 = -x_3 = -1$

$x_1 = 3x_2 + 5 = -3 + 5 = 2$

Solution set is $\{2, -1, 1\}$.

2. $\left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

x_3 is free so $x_3 = \lambda$

x_2 is free so $x_2 = \mu$

$3x_1 = 4x_2 - 2x_3 = 4\mu - 2\lambda \Rightarrow x_1 = \frac{4}{3}\mu - \frac{2}{3}\lambda$

Solution is $\begin{bmatrix} \frac{4}{3}\mu - \frac{2}{3}\lambda \\ \mu \\ \lambda \end{bmatrix} = \lambda \begin{bmatrix} -2/3 \\ 0 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 4/3 \\ 1 \\ 0 \end{bmatrix}$

3. $\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right]$ inconsistent

So $\begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$ is <u>not</u> a linear combination of

$\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$.

4. $\left[\begin{array}{ccc|c} 2 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & -4 \end{array}\right]$

$5x_3 = -4 \Rightarrow x_3 = -\frac{4}{5}$

$x_2 = -5x_3 + 2 = 4 + 2 = 6$

$x_1 = -2x_2 - 4x_3 - 2 = -12 + \frac{16}{5} - 2 = -\frac{54}{5}$

Solution is $\begin{bmatrix} -\frac{54}{5} \\ 6 \\ -\frac{4}{5} \end{bmatrix}$.

5. $\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

x_3 is free so $x_3 = \lambda$

$3x_2 = -6x_3 = -6\lambda \Rightarrow x_2 = -2\lambda$

$x_1 = -3x_2 - x_3 = 6\lambda - \lambda = 5\lambda$

The solution is $\lambda \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$.

6. $\left[\begin{array}{ccc|c} 0 & -8 & 5 & 0 \\ 3 & -7 & 4 & 0 \\ -1 & 5 & -4 & 0 \\ 1 & -3 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -1 & 5 & -4 & 0 \\ 3 & -7 & 4 & 0 \\ 0 & -8 & 5 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -8 & 5 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_3 = 0 \quad x_2 = x_3 = 0 \quad x_1 = 3x_2 - 2x_3 = 0$. So only trivial solution. Hence columns are linearly independent.

7. $\left[\begin{array}{ccc|c} 1 & 0 & -2 & -1 \\ -2 & 1 & 6 & 7 \\ 3 & -2 & -5 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & -2 & 1 & 0 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 5 & 10 \end{array}\right]$

$5x_3 = 10 \Rightarrow x_3 = 2$

$x_2 = -2x_3 + 5 = -2 \cdot 2 + 5 = 1$

$x_1 = 2x_3 - 1 = 2 \cdot 2 - 1 = 3$

The unique solution is $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

8. $\begin{bmatrix} 2 & -5 \\ 1 & 2 \\ 2 & 0 \\ 1 & 0 \end{bmatrix}$