MATH 309

Solutions to HW 6

Section 2.3

1. $A = \begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$ $\det A = -30 + 21 = -9 \neq 0$

So A is invertible.

3. $B = \begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix}$ $\det B = 5 \cdot (-7) \cdot (-1) = 35 \neq 0$

So B is invertible.

5. $C = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ -4 & -9 & 7 \end{bmatrix}$ $\det C = -3 \cdot [7 + 8] - 5[-9 + 0]$

$= -45 + 45 = 0$

So C not invertible.

Section 2.4

1. $\begin{bmatrix} I & 0 \\ E & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ EA+C & EB+D \end{bmatrix}$

5. $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$

$$\begin{bmatrix} A+BX & BY \\ C & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

$A + BX = 0 \quad X = -B^{-1}A$

$BY = I \Rightarrow Y = B^{-1}$

$C = Z \quad Z = C$

$0 = 0$

9. $\begin{bmatrix} I & 0 & 0 \\ X & I & 0 \\ Y & 0 & I \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \\ 0 & B_{32} \end{bmatrix}$

$$\begin{bmatrix} A_{11} & A_{12} \\ XA_{11}+A_{21} & XA_{12}+A_{22} \\ YA_{11}+A_{31} & YA_{12}+A_{32} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \\ 0 & B_{32} \end{bmatrix}$$

$A_{11} = B_{11}$

$A_{12} = B_{12}$

$XA_{11} + A_{21} = 0 \quad X = -A_{21}A_{11}^{-1}$

$XA_{12} + A_{22} = B_{22}$

$YA_{11} + A_{31} = 0 \quad Y = -A_{31}A_{11}^{-1}$

$YA_{12} + A_{32} = B_{32}$

$B_{22} = XA_{12} + A_{22} = [-A_{21}A_{11}^{-1}]A_{12} + A_{22}$

Section 2.5

4. $A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix} = L \cdot U$

$\underset{=}{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$

$[L \; \underset{=}{b}] = \begin{bmatrix} 1 & 0 & 0 & -7 \\ -1 & 1 & 0 & 5 \\ 2 & -5 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -2 \\ 0 & -5 & 1 & 16 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 6 \end{bmatrix}$

So $\underset{=}{y} = \begin{bmatrix} -7 \\ -2 \\ 6 \end{bmatrix}$

$[U \; \underset{=}{y}] = \begin{bmatrix} 3 & -7 & -2 & -7 \\ 0 & -2 & -1 & -2 \\ 0 & 0 & -1 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -\frac{7}{3} & -\frac{2}{3} & -\frac{7}{3} \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 1 & -6 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -6 \end{bmatrix}$

So $\underset{=}{x} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$

3. $A = \begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ 0 & -3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

$\underline{b} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

$[L \; \underline{b}] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -3 & 1 & 0 & 0 \\ 4 & -1 & 1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & -1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

So $\underline{y} = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$

$[U \; \underline{y}] = \begin{bmatrix} 2 & -1 & 2 & 1 \\ 0 & -3 & 4 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & \frac{2}{3} & 0 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

So $\underline{x} = \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}$

9. $\begin{bmatrix} 3 & -1 & 2 \\ -3 & -2 & 10 \\ 9 & -5 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -1 & 2 \\ 0 & -3 & 12 \\ 0 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -1 & 2 \\ 0 & -3 & 12 \\ 0 & 0 & -8 \end{bmatrix}$

$U = \begin{bmatrix} 3 & -1 & 2 \\ 0 & -3 & 12 \\ 0 & 0 & -8 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2\frac{1}{3} & 1 \end{bmatrix}$

11. $\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix} \to \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 5 & 1 \end{bmatrix} \to \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix} = U$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix}$$

Section 2.8

5. $\left[\begin{array}{cc|c} 2 & -4 & 8 \\ 3 & -5 & 2 \\ -5 & 8 & -9 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & -4 & 8 \\ 0 & 1 & -10 \\ 0 & -2 & 11 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & -4 & 8 \\ 0 & 2 & -10 \\ 0 & 0 & -9 \end{array}\right]$

inconsistent

So $\underline{w}$ is not a linear combination of $\underline{v}_1, \underline{v}_2$.

6. $\left[\begin{array}{ccc|c} 1 & 4 & 5 & -4 \\ -2 & -7 & -8 & 10 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 5 & -5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & -7 & -14 & 9 \\ 0 & -5 & -10 & 7 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 23 \\ 0 & 0 & 0 & 17 \end{array}\right]$

inconsistet

So $\underline{u}$ is not a linear combination of $\underline{v}_1, \underline{v}_2, \underline{v}_3$.

7. a) These are three vectors

b) $\left[\begin{array}{ccc|c} 2 & -3 & -4 & 0 \\ -8 & 8 & 6 & 0 \\ 6 & -7 & -7 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & -3 & -4 & 0 \\ 0 & -4 & -10 & 0 \\ 0 & 2 & 5 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 2 & -3 & -4 & 0 \\ 0 & -4 & -10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$c_3 = \lambda$

$c_2 = -\frac{5}{2} c_3 = -\frac{5}{2}\lambda$

$2c_1 = 3c_2 + 4c_3$

$= -\frac{15}{2}\lambda + 4\lambda = -\frac{7}{2}\lambda$

So $(c_1, c_2, c_3) = \lambda\left(-\frac{7}{2}, -\frac{5}{2}, 1\right)$,

The vectors are linearly dependent. So

$\dim \operatorname{Span}\{\underline{v}_1, \underline{v}_2, \underline{v}_3\} = 2$

There are 3 vectors in Col A but the space they span has dimension 2

c) $\left[\begin{array}{ccc|c} 2 & -3 & -4 & 6 \\ -8 & 8 & 6 & -10 \\ 6 & -7 & -7 & 11 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & -3 & -4 & 6 \\ 0 & -4 & -10 & 14 \\ 0 & 2 & 5 & -7 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 2 & -3 & -4 & 6 \\ 0 & -4 & -10 & 14 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$c_3 = \lambda$

$c_2 = -\frac{5}{2} c_3 \quad -\frac{7}{2} = -\frac{5}{2}\lambda - \frac{7}{2}$

$c_1 = \frac{3}{2} c_2 + 2c_3 + 3 = -\frac{15}{4}\lambda - \frac{21}{4} + 2\lambda + 3 = -\frac{7}{4}\lambda - \frac{9}{4}$

So $\underline{p}$ is in Col A.