Math 233
Krantz

Spring, 2017
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First Midterm

General Instructions: Read the statement of each problem carefully. Do only what is requested—nothing more and nothing less. Of course you need not show any work for the multiple choice or the TRUE/FALSE questions. For the questions that require a written answer, provide a *complete solution.* If you only write the answer then you will not get full credit.

Be sure to ask questions if anything is unclear. This exam is worth 100 points.

(10 points) **1.** Let $\mathbf{v} = \langle 2, -1, 4\rangle$ and $\mathbf{w} = \langle 3, 1, -6\rangle$. Then $2\mathbf{v} - 4\mathbf{w}$ equals

(a) $\langle -8, -6, 32\rangle$

(b) $\langle 8, 6, 32\rangle$

(c) $\langle 6, 8, -32\rangle$

(d) $\langle 8, 6, -32\rangle$

(e) none of the above

$2\underset{=}{v} - 4\underset{=}{w} = \langle 4, -2, 8\rangle - \langle 12, 4, -24\rangle$

$= \langle -8, -6, 32\rangle$

(10 points) **2.** The equation for the sphere in 3-space with center $(2, 1, 5)$ and radius 3 is

(a) $(x+2)^2 + (y+1)^2 + (z+5)^2 = 9$

(b) $(x+2)^2 + (y-1)^2 + (z+5)^2 = 16$

(c) $(x-2)^2 + (y-1)^2 + (z-5)^2 = 25$

(d) $(x-2)^2 + (y-1)^2 + (z-5)^2 = 9$

(e) $(x+2)^2 + (y+1)^2 + (z-5)^2 = 9$

$$(x-2)^2 + (y-1)^2 + (z-5)^2 = 3^2$$

(10 points) **3.** The projection of the vector $\mathbf{v} = \langle -4, 1, 2\rangle$ onto the vector $\mathbf{w} = \langle 1, 1, 2\rangle$ is

(a) $\langle 2, 2, 4\rangle$

(b) $\langle 1/2, 1/2, 1\rangle$

(c) $\langle -1/6, -1/6, -1/3\rangle$

(d) $\langle 1/6, 1/6, 1/2\rangle$

(e) $\langle 1/6, 1/6, 1/3\rangle$

$$P_{\underline{w}}(\underline{v}) = \left(\frac{\underline{v}\cdot\underline{w}}{\underline{w}\cdot\underline{w}}\right)\underline{w} = \left(\frac{-4+1+4}{1+1+4}\right)\langle 1, 1, 2\rangle$$

$$= \frac{1}{6}\langle 1, 1, 2\rangle$$

$$= \left\langle \frac{1}{6}, \frac{1}{6}, \frac{1}{3}\right\rangle$$

(10 points) **4.** Which pairs of the vectors $\mathbf{u} = \langle 4, -2, 1\rangle$, $\mathbf{v} = \langle 1, 2, 0\rangle$, and $\mathbf{w} = \langle 2, 4, 8\rangle$ are perpendicular?

(a) **u** and **w**
(b) **v** and **w**
(c) **u** and **v**
(d) **u** and **u**
(e) **w** and **u**

$\underline{u} \cdot \underline{v} = 4 - 4 + 0 = 0$

$\underline{u} \cdot \underline{w} \neq 0$

$\underline{v} \cdot \underline{w} \neq 0$

(10 points) **5.** **TRUE** or **FALSE**: The cross product of two vectors **v** and **w** is perpendicular to **v** and parallel to **w**.

(a) **TRUE**
(b) **FALSE**

the cross product of $\underline{v}$ and $\underline{w}$ is perpendicular to $\underline{v}$ and perpendicular to $\underline{w}$

(10 points) **6.** The angle between the vectors $\mathbf{v} = \langle 1, 1, 0 \rangle$ and $\mathbf{w} = \langle 0, 1, 1 \rangle$ is

(a) $\pi/4$
(b) $\pi/2$
(c) $\pi/3$
(d) $\pi/6$
(e) $2\pi/3$

$$\cos\theta = \frac{\underline{v} \cdot \underline{w}}{\|\underline{v}\| \|\underline{w}\|} = \frac{0 + 1 + 0}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$$

So $\theta = \frac{\pi}{3}$

(10 points) **7.** The cross product of the vectors $\mathbf{v} = \langle 5, 1, 2 \rangle$ and $\mathbf{w} = \langle -2, 1, -3 \rangle$ is

(a) $\langle -5, 11, 7 \rangle$
(b) $\langle 5, 11, 7 \rangle$
(c) $\langle -5, -11, 7 \rangle$
(d) $\langle -5, 11, -7 \rangle$
(e) $\langle 5, 11, 7 \rangle$

$$\underline{v} \times \underline{w} = \det \begin{pmatrix} \underline{\hat{i}} & \underline{\hat{j}} & \underline{\hat{k}} \\ 5 & 1 & 2 \\ -2 & 1 & -3 \end{pmatrix} = \underline{\hat{i}}(-5) - \underline{\hat{j}}(-11) + \underline{\hat{k}}\,7$$

$$= \langle -5, 11, 7 \rangle$$

(10 points) **8.** The equation of the plane through the point $(2, 5, 3)$ and perpendicular to the vector $\mathbf{n} = \langle 1, 3, 1 \rangle$ is

(a) $x - 3y + z = 20$

(b) $x + 3y + z = 20$

(c) $x + 3y - z = 20$

(d) $-x + 3y + z = 20$

(e) $-x - 3y - z = 20$

$\langle 1, 3, 1 \rangle \cdot \langle x-2, y-5, z-3 \rangle = 0$

$(x-2) + (3y-15) + (z-3) = 0$

$x + 3y + z = 20$

(10 points) **9.** Specify a point that lies on the line

$$\begin{aligned} x &= 3t+2 \\ y &= 2t-5 \\ z &= t+4. \end{aligned}$$

Also write a vector that is parallel to this line.

Setting $t=0$ gives the point $(2,-5,4)$.

The direction vector for the line is $\langle 3, 2, 1 \rangle$.

(10 points) **10.** Find the point of intersection of the plane $x + 2y + 4z = 18$ and the line

$$\begin{aligned} x &= 2t+2 \\ y &= t+2 \\ z &= t+1. \end{aligned}$$

$(2t+2) + 2(t+2) + 4(t+1) = 18$

$2t + 2 + 2t + 4 + 4t + 4 = 18$

$8t = 8$

$t = 1$

So $x = 4, y = 3, z = 2$ is the point of intersection.