Math 217 Krantz

PRACTICE THIRD MIDTERM EXAM

1. Verify that 0 is a regular singular point for the differential equation

$$x^2y'' + x(x+1)y' - y = 0\,.$$

Now determine the values of m that will make the Frobenius solution valid.

2. Calculate the Fourier series of the function $f(x) = -x$, $-\pi \leq x \leq \pi$.

3. Calculate the cosine series of the function $g(x) = 2x$ on the interval $[0, \pi]$.

4. Calculate the Fourier series of the function $h(x) = -x$ on the interval $[-3, 3]$.

5. Calculate the inverse Laplace transform of the function

$$F(p) = \frac{1}{p^4 - 4p^2}\,.$$

6. Solve the initial value problem

$$y'' - 2y' + y = 0 \quad , \quad y(0) = 0\ ,\ y'(0) = 3\,.$$

using the Laplace transform.

7. Calculate the convolution of the functions $f(t) = t$ and $g(t) = e^{2t}$.

8. Use the unit impulse function to solve the initial value problem

$$y'' + y' - 6y = t \quad , \quad y(0) = 0\ ,\ y'(0) = 0\,.$$

9. Explain why the function $f(x) = (\cos x) \cdot (\sin x)$ is odd on the interval $[-\pi, \pi]$.

10. Define an inner product on the square integrable functions on $[-\pi, \pi]$ by

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot g(x)\, dx\,.$$

Using this inner product, explain why the functions $\sin jx$ and $\cos kx$ are orthogonal for integers j and k.

11. If $F(p)$ is the Laplace transform of the function f, then explain why

$$\frac{d}{dp} F(p) = L[-xf(x)]\,.$$

12. Calculate

$$L[xe^x]\,.$$