Math 217 Spring, 2024
Krantz

FINAL EXAM

1. The solution of the initial value problem

$$\begin{aligned} y' + (1/x)y &= x \\ y(1) &= 1 \end{aligned}$$

is what?

2. The equation

$$(xy - x^3)dx + x^2 dy = 0$$

is *not* exact. Find an integrating factor for this equation.

3. Use the method of reduction of order to solve the differential equation

$$xy'' - y' = x\,.$$

4. Find the solution of the following initial value problem:

$$y'' - 5y' + 6y = 0; \qquad y(0) = 1, \quad y'(0) = 1\,.$$

5. The equation $xy'' - y' = 0$ has the solution $y_1(x) = x^2$. Find a second linearly independent solution y_2 and then find the general solution.

6. Use the method of undetermined coefficients to find the general solution of the differential equation

$$y'' - 3y' + 2y = x\,.$$

7. Find a power series solution of the form $\sum_j a_j x^j$ for the following ODE:

$$y' = y\,.$$

Identify the function it represents.

8. Calculate the Fourier series of the function $f(x) = x$, $-\pi \leq x \leq \pi$.

9. Use the unit impulse function to solve the initial value problem

$$y'' + 3y' - 4y = 2t \quad , \quad y(0) = 0 \ , \ y'(0) = 0 \, .$$

10. Find the eigenvalues λ_n and eigenfunctions y_n for the equation $y'' + \lambda y = 0$ on the interval $[0, \pi/2]$ with the boundary conditions $y(0) = 0$, $y(\pi/2) = 0$.

11. Solve the Dirichlet problem on the unit disc with boundary function $f(\theta)$ equaling 2 on the top half of the boundary and equaling 1 on the bottom half of the boundary.

12. Consider the heat equation for $z(x,t)$ on $t \geq 0$ and $0 \leq x \leq \pi$:

$$\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2},$$

with boundary conditions $z(0,t) = 0$ and $z(\pi,t) = 0$ for all t. Find the solution if the initial temperature is given by

$$z(x,0) = -\sin x + 6 \sin x \cos x, \qquad 0 \leq x \leq \pi.$$