Supplementary problem on rotation of axes

Read the supplementary example on Rotation of Axes (distibruted in class last week and posted online). Then solve the following problem.

1. Let ϕ be the angle for which $\begin{cases} \cos\phi = \frac{2}{\sqrt{5}} \\ \sin\phi = -\frac{1}{\sqrt{5}} \end{cases}$.

Then $\tan\phi = \frac{\sin\phi}{\cos\phi} = -\frac{1}{2}$. The <u>exact</u> value of ϕ is $\arctan(-1/2)$, so the <u>approximate</u> value of ϕ (*from a calculator*) is -0.4636 radians or $-26.5651°$.

a) What is the matrix A that represents a rotation around the origin through the angle ϕ ? (*Since ϕ is negative, the rotation is <u>clockwise</u> around the origin.*)

b) Explain why the <u>columns</u> $\boldsymbol{b_1}$ and $\boldsymbol{b_2}$ of A are a basis for $\mathbb{R}^2$.

The vectors $\boldsymbol{b_1}$ and $\boldsymbol{b_2}$ *determine a new set of coordinate axes x'-y' for $\mathbb{R}^2$. The standard x-y coordinate system is rotated by ϕ to get the new x'-y' coordinate axes.*

*This matrix A is also the matrix P_B that connects x'-y' coordinates to the standard x-y coordinates. Look at the comments "**Important Observation: Two Points of View**" at the end of the example on rotation of axes*

c) Draw the graph of $8x^2 - 4xy + 5y^2 = 1$ <u>by</u> first converting the equation into x'-y' coordinates and then drawing the graph with respect to the new x'-y' coordinate system. *Indicate in your picture the original x-y coordinate axes, the new x'-y'axes and indicate the angle ϕ in your drawing.*