Summary Theorem About Diagonalization

Suppose A is an $n \times n$ matrix with eigenvalues
$\lambda_1, ..., \lambda_i, \ ..., \ \lambda_p$

1) A is diagonalizable if and only if there is a basis of eigenvectors $\{\boldsymbol{v_1}, ..., \ \boldsymbol{v_i}, ..., \boldsymbol{v_n}\}$ for $\mathbb{R}^n$.

Then $A = PDP^{-1}$, where

$$P = [\boldsymbol{v_1} \ ... \ \boldsymbol{v_i} \ ... \ \boldsymbol{v_n}]$$

D is diagonal matrix with the eigenvalues
$\lambda_1, ..., \lambda_i, ..., \lambda_n$ on its diagonal

2) If we choose a basis for each eigenspace $\mathcal{B}_1$,..., $\mathcal{B}_p$ and unite these bases into one collection $\mathcal{B}$, then $\mathcal{B}$ is a linearly independent set of eigenvectors in $\mathbb{R}^n$ – so $\mathcal{B}$ is a basis for $\mathbb{R}^n$ if and only if the sum of the dimensions of the eigenspaces is n.

3) For each individual eigenvalue λ_i :

$$\dim(\text{eigenspace}) \leq \text{ multiplicity of eigenvalue}$$