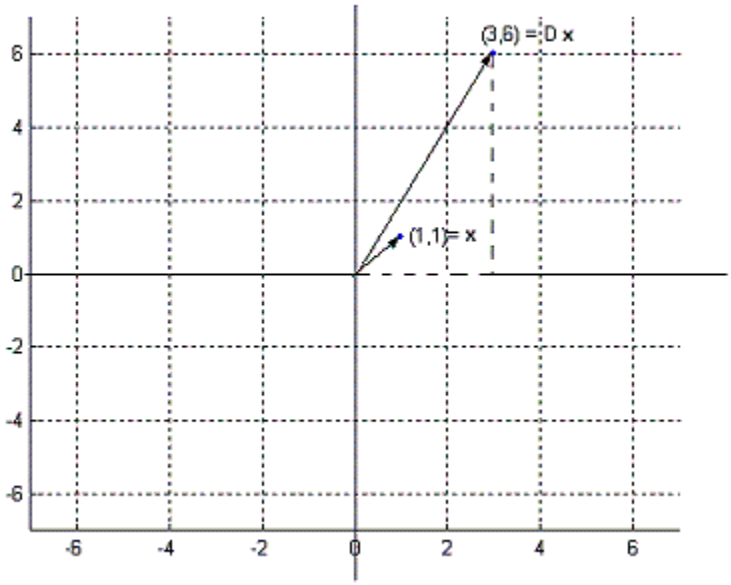
$$T : \mathbb{R}^2 \to \mathbb{R}^2 \,,\; T(\boldsymbol{x}) = D\boldsymbol{x} = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \boldsymbol{x}.$$

$$D\mathbf{e_1} = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} = 3\boldsymbol{e_1}$$

$$D\mathbf{e_2} = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} = 6\boldsymbol{e_2}\,,$$

$$D\boldsymbol{x} = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 \\ 6x_2 \end{bmatrix}$$

$$\mathrm{D} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \;:$$



$T(\boldsymbol{x}) = A\boldsymbol{x}$, where $A = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$

somehow!?!

$$\downarrow \quad A = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$= \quad P \quad D \quad P^{-1}$$

Let basis $\mathcal{B} = \{\boldsymbol{b_1}, \boldsymbol{b_2}\} = \{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \}$

Then $P = P_{\mathcal{B}} =$ change of coordinates matrix

$$P_{\mathcal{B}}[\boldsymbol{x}]_{\mathcal{B}} = \boldsymbol{x}$$

For $\boldsymbol{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Then

$$A\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}.$$

<u>or can calculate</u>

$$A\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}$$

$$P_{\mathcal{B}} \quad D \quad P_{\mathcal{B}}^{-1} \quad \boldsymbol{x}$$

$A\boldsymbol{x} =$

$$\underset{P_{\mathcal{B}}}{\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}} \underset{D}{\begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix}} \underset{P_{\mathcal{B}}^{-1}}{\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}} \underset{\boldsymbol{x}}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}$$

$$\underset{\text{Convert to } \mathcal{B} \text{ coordinates}}{\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{4}{3} \end{bmatrix}.$$

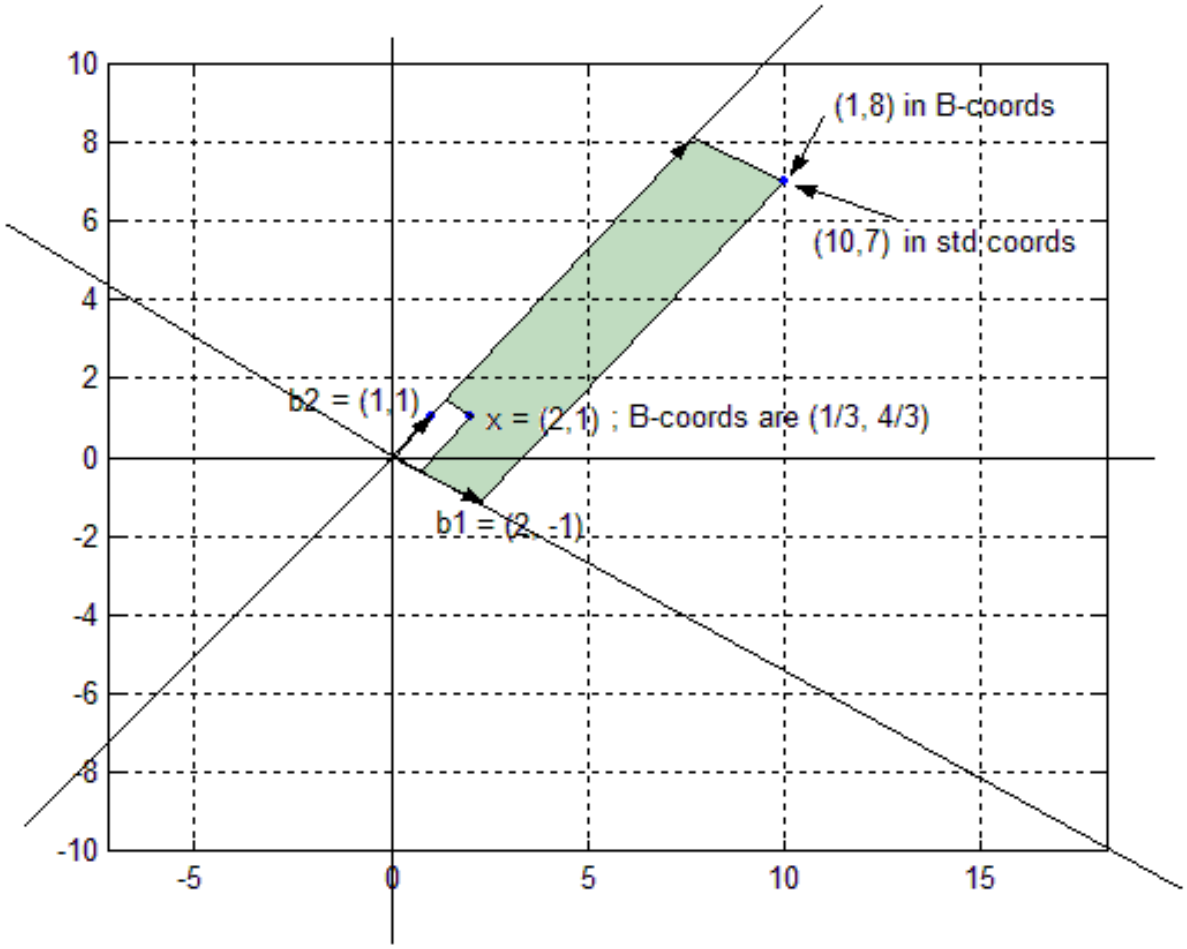
Then $\begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{4}{3} \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$

Rescale $\mathcal{B}$ coordinates

Then $\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}$

Convert back to standard coordinates

(*see following diagram*)


(1,8) in B-coords
(10,7) in std coords
b2 = (1,1)
x = (2,1) ; B-coords are (1/3, 4/3)
b1 = (2, -1)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
0
5
10
15